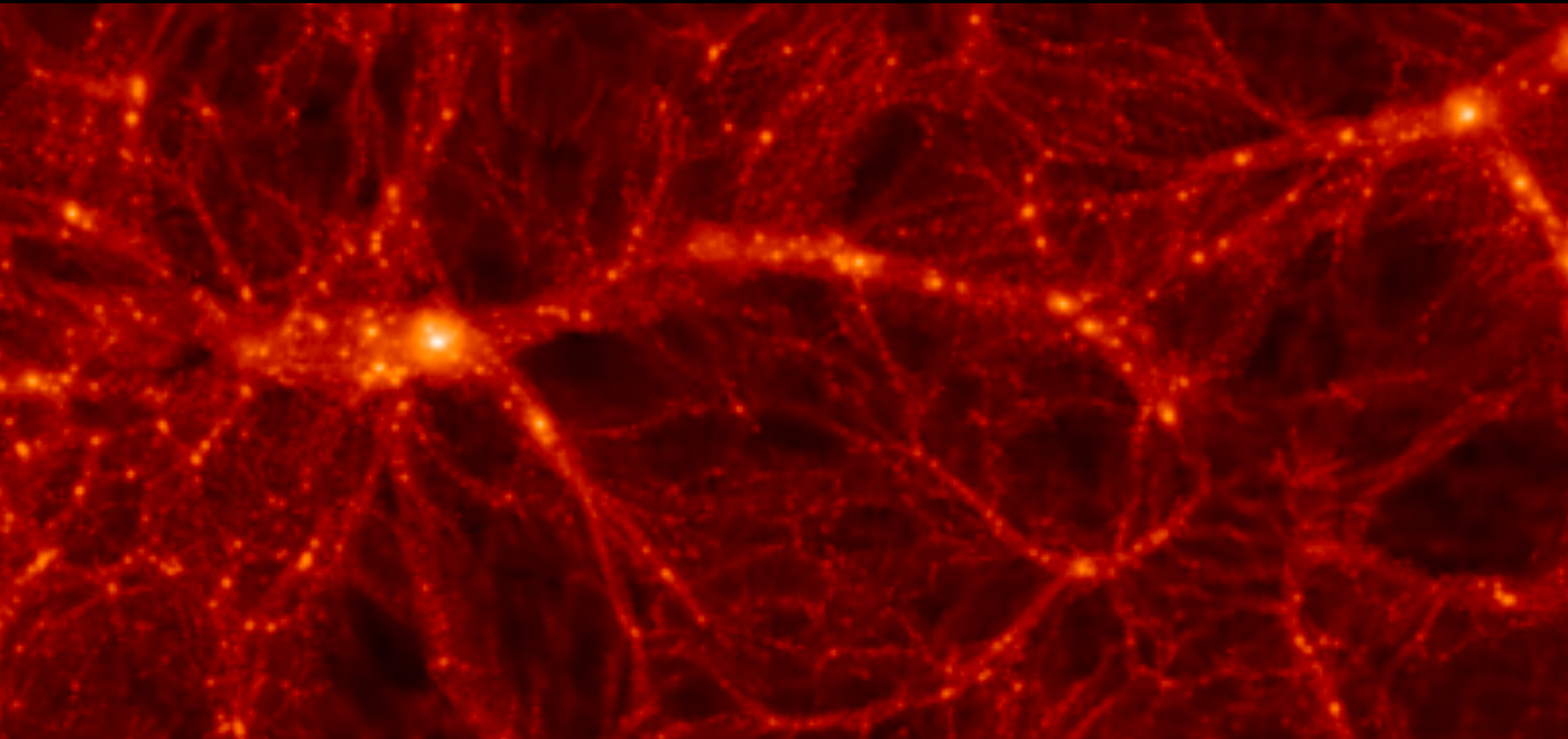
AN INTRODUCTION TO MODERN COSMOLOGY



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ISAPP 2009, 28 JULY 2009

A rough outline of the lectures:

- Basic cosmology – the Friedmann equation
- Thermodynamics and particle decoupling in the early universe
- Big Bang Nucleosynthesis
- Structure formation in the universe
- Using structure formation to probe cosmological parameters

Friedmann-Robertson-Walker Cosmology

Line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Reduces to

$$ds^2 = -dt^2 + a^2(t)\left\{dr^2 + S_k^2(r)d\Omega^2\right\}$$

in a homogeneous and isotropic universe

$$S_k^2(r) = \begin{cases} \sin^2(r) & k = 1 \\ r^2 & k = 0 \\ \sinh^2(r) & k = -1 \end{cases}$$

a(t): Scale factor, only dynamical variable

The Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

is a combination of 10 coupled differential equations since the involved tensors are explicitly symmetric

However, it reduces to an evolution equation for *a(t)* (The Friedmann equation) in a homogeneous and isotropic universe

$$\boxed{H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{a^2} = \frac{8\pi G\rho_{TOT}}{3} - \frac{k}{a^2}}$$

THE TOTAL ENERGY DENSITY THEN BY DEFINITION INCLUDES NON-RELATIVISTIC MATTER, RADIATION AND THE COSMOLOGICAL CONSTANT

$$\boxed{\rho_{TOT} = \rho_{MATTER} + \rho_{RADIATION} + \rho_{\Lambda}}$$

HOWEVER, THESE TYPES OF ENERGY BEHAVE COMPLETELY DIFFERENTLY AS A FUNCTION OF TIME AND SCALE FACTOR

$$\rho_{MATTER} \propto n \times m \propto a(t)^{-3}$$

$$\rho_{RADIATION} \propto n \times \lambda^{-1} \propto a(t)^{-4}$$

$$\rho_{\Lambda} \propto \Lambda \propto \mathrm{Constant}$$

FROM THE ABOVE EQUATION

$$\rho_{\mathrm{RADIATION}} \sim a^{-4}(t)$$

AND THE FACT THAT

$$\rho_{\mathrm{RADIATION}} \sim T^{4}$$

IT CAN BE SEEN THAT THE EFFECTIVE 'TEMPERATURE' OF RADIATION SCALES AS

$$T \propto a^{-1}$$

A DEFINITION:

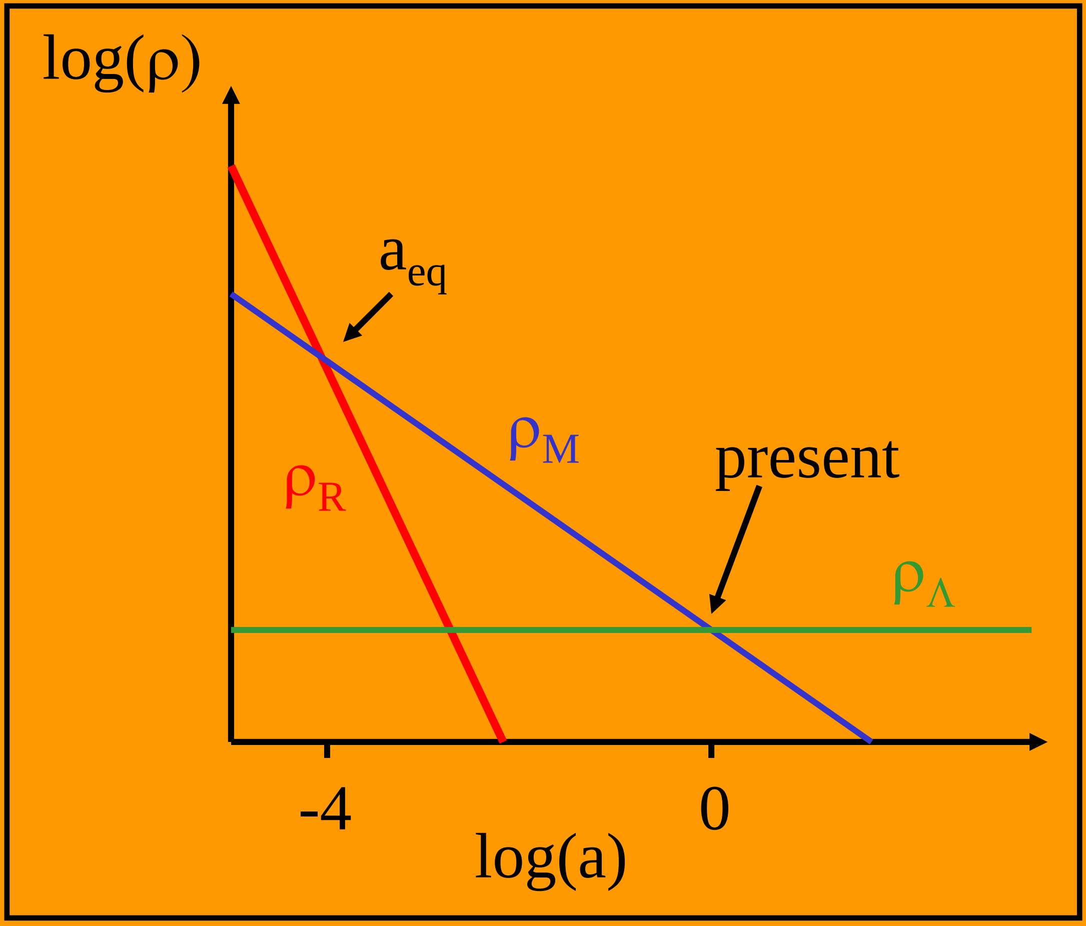
A QUANTITY WHICH IS CONSISTENTLY USED IS THE REDSHIFT, DEFINED AS

$$1 + z \equiv \frac{a_0}{a}$$

FROM THE SCALING OF PHOTON ENERGY IT CAN IMMEDIATELY BE SEEN THAT THE OBSERVED WAVELENGTH OF A PHOTON IS RELATED TO THE SCALE FACTOR OF THE UNIVERSE WHEN IT WAS EMITTED

$$\frac{\lambda_{\mathrm{OBSERVED}}}{\lambda_{\mathrm{EMITTED}}} = 1 + z$$

EVOLUTION OF ENERGY DENSITY WITH SCALE FACTOR



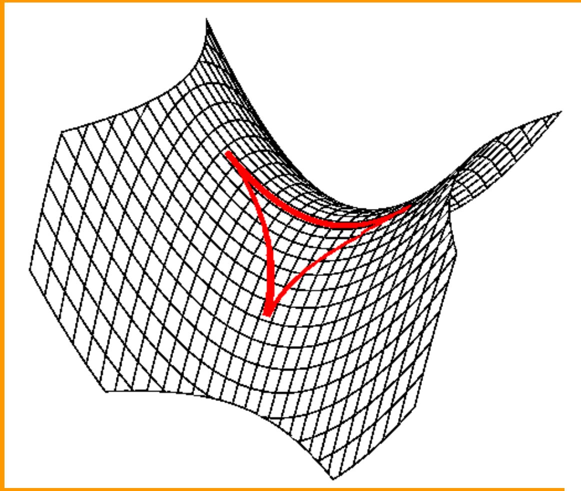
The geometry of the universe

The Friedmann equation can be recast in terms of the density parameter, Ω

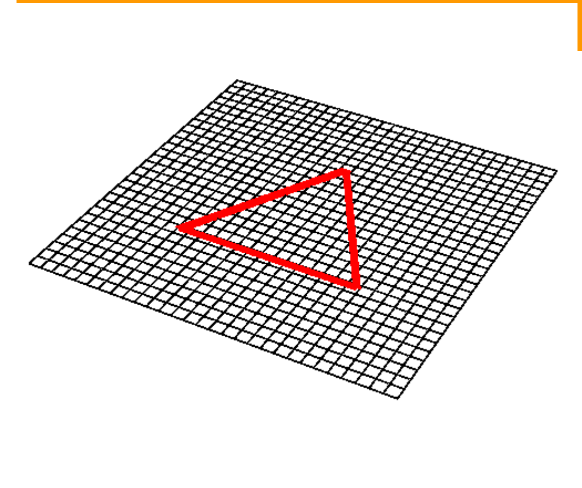
$$\frac{k}{H^2a^2} = \Omega_M + \Omega_\Lambda + \Omega_R - 1 = \Omega_{\mathrm{TOT}} - 1$$

$$\Omega \equiv \frac{8\pi G\rho}{3H^2}$$

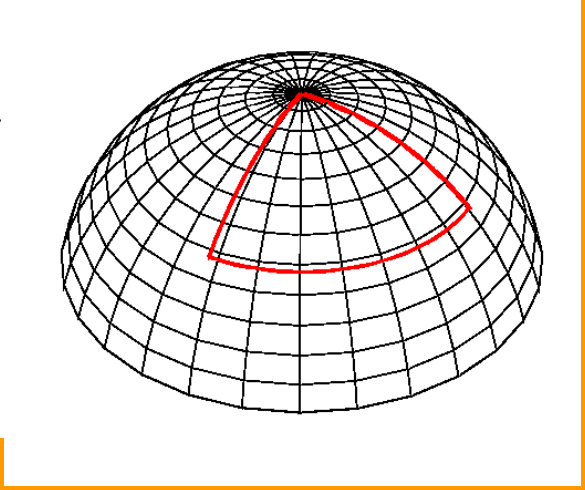
Open Universe
k = -1, Ω < 1



Flat Universe
k = 0, Ω =1



Closed Universe
k =1, Ω >1



An empty universe expands linearly with time

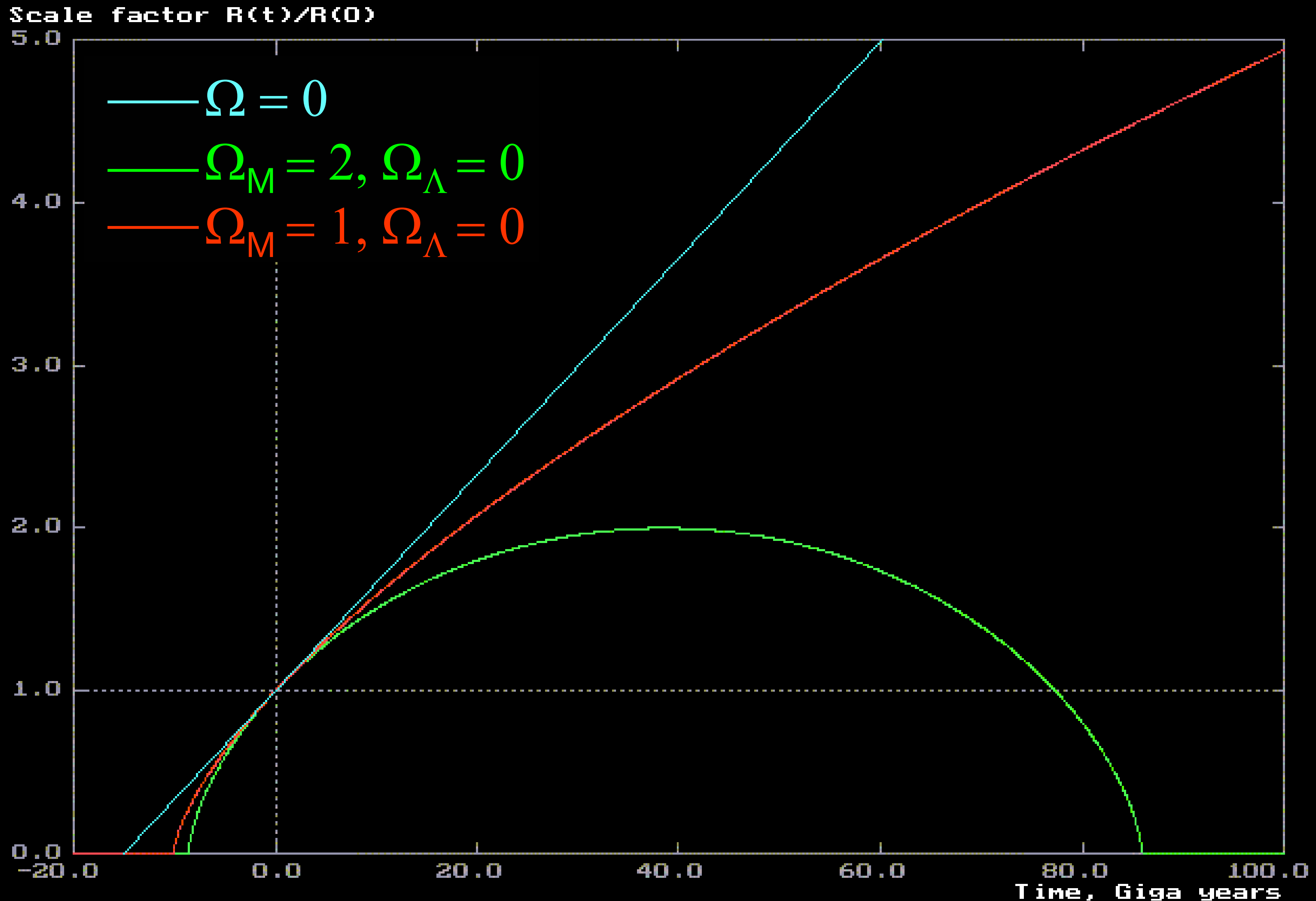
$$\boxed{a(t) \sim t}$$

Matter acts to slow the expansion, for example

$$\boxed{a(t) \sim t^{1/2} \text{ for } \Omega_R = 1, \Omega_M = 0}$$

$$\boxed{a(t) \sim t^{2/3} \text{ for } \Omega_R = 0, \Omega_M = 1}$$

If $\Omega_M + \Omega_R > 1$ then the universe eventually recollapses



Scale factor R(t)/R(0)
5.0
4.0
3.0
2.0
1.0
0.0
$\Omega = 0$
$\Omega_M = 2, \Omega_\Lambda = 0$
$\Omega_M = 1, \Omega_\Lambda = 0$
-20.0
0.0
20.0
40.0
60.0
80.0
100.0
Time, Giga years

A cosmological constant acts to accelerate the expansion.

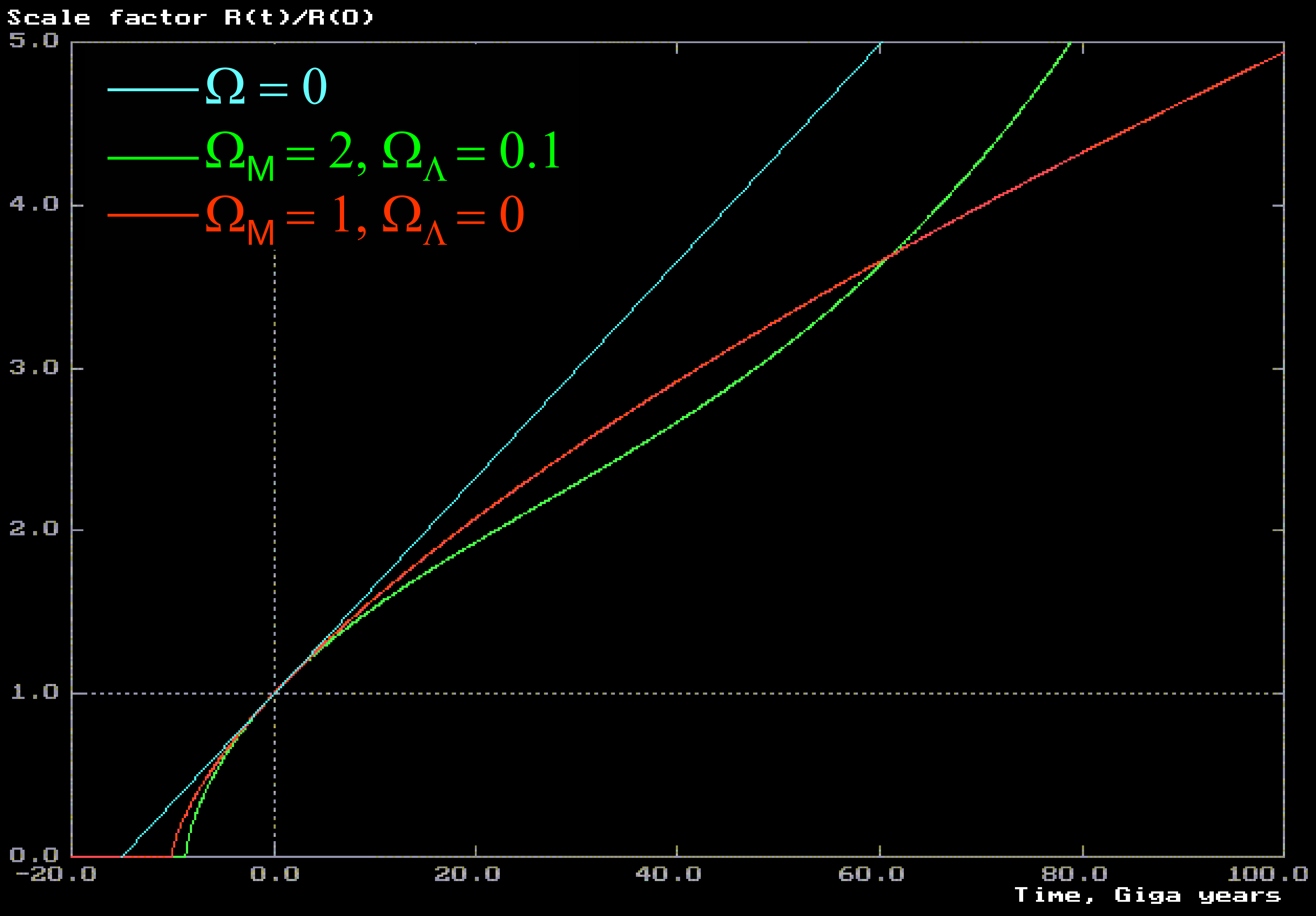
$$\boxed{a(t) \sim e^{\sqrt{\Lambda}t} \text{ for } \Omega_\Lambda = 1, \Omega_M = 0}$$

In general the pressure of an energy density component can be written as

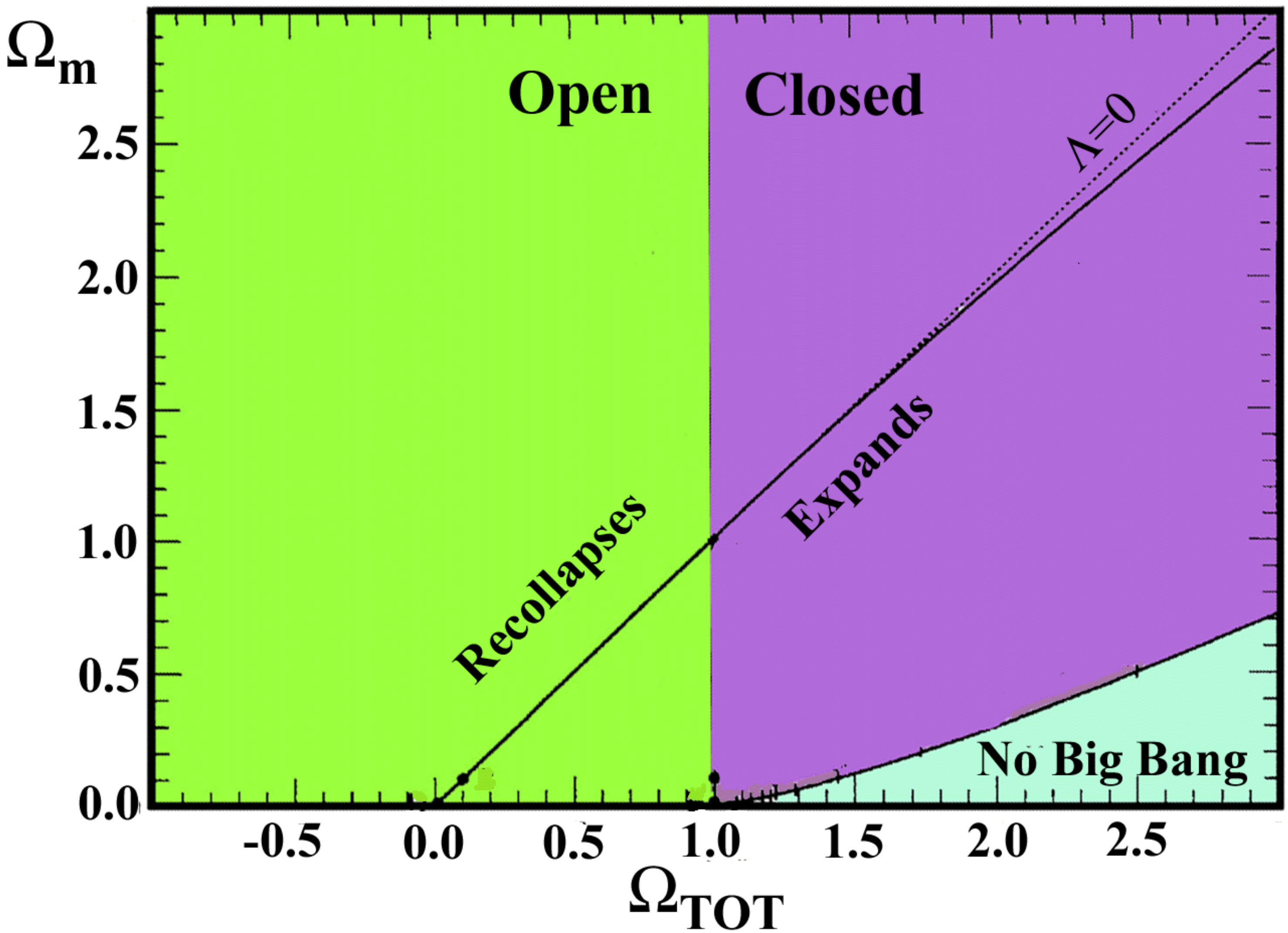
$$\boxed{P = w\rho}$$

For the cosmological constant, *w* = -1

Any component which has *w* < -1/3 leads to an accelerated expansion and is referred to as <u>dark energy</u>



Scale factor R(t)/R(0)
Ω = 0
Ω_M = 2, Ω_Λ = 0.1
Ω_M = 1, Ω_Λ = 0
5.0
4.0
3.0
2.0
1.0
0.0
-20.0
0.0
20.0
40.0
60.0
80.0
100.0
Time, Giga years


Ω_m
Open
Closed
Λ=0
2.5
2.0
1.5
1.0
0.5
0.0
Expands
Recollapses
No Big Bang
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
Ω_{TOT}

A BIT OF EARLY UNIVERSE COSMOLOGY

Thermodynamics in the early universe

In equilibrium, distribution functions have the form

$$f_{EQ} = \frac{1}{\exp((E-\mu)/T \pm 1)} \quad , \quad E = \sqrt{p^2 + m^2}$$

When $m \sim T$ particles disappear because of Boltzmann supression

$$f \rightarrow f_{MB} = e^{-(m-\mu)/T} e^{-p^2/2mT}$$

Decoupled particles: If particles are decoupled from other species their comoving number density is conserved. The momentum redshifts as $p \sim 1/a$

The entropy density of a species with MB statistics is given by

$$s = -\int f \ln f \; d^3 p \quad , \quad f = e^{-(E-\mu)/T}$$

In equilibrium, $\mu(X) = -\mu(\bar{X})$

(true if processes like $X\bar{X} \leftrightarrow \gamma\gamma$ occur rapidly)

This means that entropy is maximised when

$$\mu(X) = -\mu(\bar{X}) = 0$$

In equilibrium neutrinos and anti-neutrinos are equal in number!
However, the neutrino lepton number is not nearly as well constrained observationally as the baryon number

It is possible that $$\frac{n_\nu}{n_\gamma} >> \frac{n_B}{n_\gamma} \sim 10^{-10}$$

Thermal evolution after the end of inflation

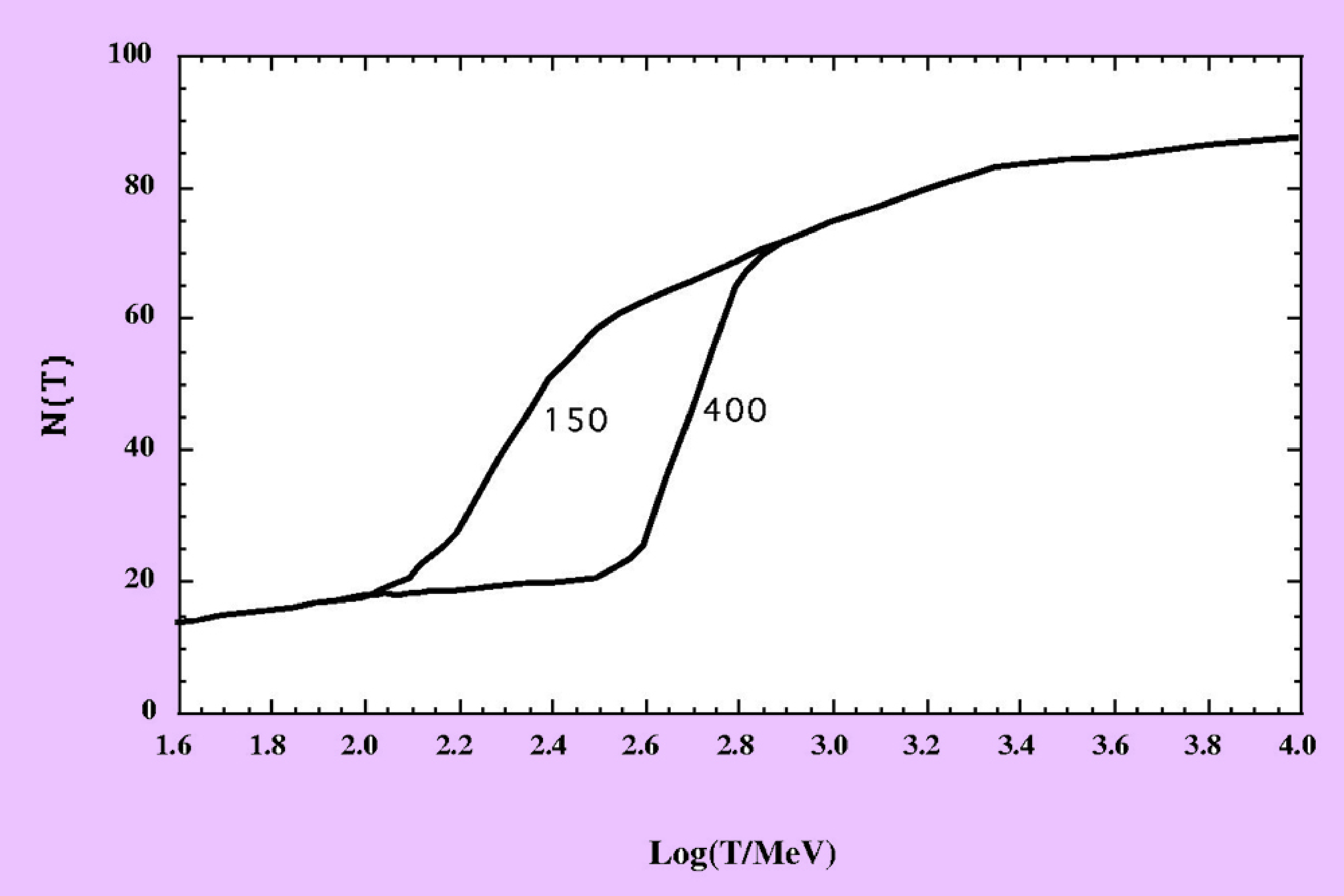
$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3}$$

$$\rho_B = \frac{\pi^2}{30} g T^4 \quad \text{for bosons}$$

$$\rho_F = \frac{7}{8}\frac{\pi^2}{30} g T^4 \quad \text{for fermions}$$

Total energy density

$$\rho_{TOT} = (\sum g_B + \frac{7}{8}\sum g_F)\frac{\pi^2}{30}T^4 = \frac{\pi^2}{30}N(T)T^4$$



Temperature evolution of *N*(*T*)

In a radiation dominated universe the time-temperature relation is then of the form

$$t = \frac{1}{2} H^{-1} = \left(\frac{3}{32\pi G\rho} \right)^{1/2} \Rightarrow t_s \approx 2.4 \left[N(T) \right]^{-1/2} T_{MeV}^{-2}$$

The number and energy density for a given species, *X*, is given by the Boltzmann equation

$$\frac{\partial f_X}{\partial t} + pH\frac{\partial f_X}{\partial p} = C_e[f_X] + C_i[f_X]$$

$C_e[f]$: *Elastic collisions*, conserves particle number but energy exchange possible (e.g. $X + i \rightarrow X + i$) [scattering equilibrium]

$C_i[f]$: *Inelastic collisions*, changes particle number (e.g. $X + \bar{X} \rightarrow i + \bar{i}$) [chemical equilibrium]

Usually, $C_e[f] >> C_i[f]$ so that one can assume that elastic scattering equilibrium always holds.

If this is true, then the form of *f* is always Fermi-Dirac or Bose-Einstein, but with a possible chemical potential.

Particle decoupling

The inelastic reaction rate per particle for species X is

$$\Gamma_{\text{int}} = \int C_i\left[f_X\right]\frac{d^3 p_X}{(2\pi)^3} = n_X \langle \sigma v \rangle$$

In general, a species decouples from chemical equlibrium when

$$\Gamma_{\text{int}} \approx H$$

$$H \approx 2N(T)^{1/2}\frac{T^2}{m_{Pl}}$$

The prime example is the decoupling of light neutrinos ($m < T_D$)

$$\Gamma_{weak} = n\langle \sigma v \rangle \approx T^3 G_F^2 T^2 \Rightarrow T_D \approx 1\,\mathrm{MeV}$$

After neutrino decoupling electron-positron annihilation takes place (at T~m_e/3)

Entropy is conserved because of equilibrium in the e^+- e^-- γ plasma and therefore

$$s_i = s_f \;\Rightarrow\; (2 + 4\frac{7}{8})T_i^3 = 2T_f^3 \;\Rightarrow\; \frac{T_f}{T_i} = \left(\frac{11}{4}\right)^{1/3}$$

The neutrino temperature is unchanged by this because they are decoupled and therefore

$$T_\nu = (4/11)^{1/3} T_\gamma \approx 0.71 T_\gamma \quad \text{(after annihilation)}$$

BIG BANG NUCLEOSYNTHESIS

The baryon number left after baryogenesis is usually expressed in terms of the parameter η

$$\eta \equiv \left.\frac{n_B}{n_\gamma}\right|_{t=t_0}$$

According to observations $\eta \sim 10^{-10}$ and therefore the parameter

$$\eta_{10} \equiv 10^{10} \times \eta$$

is often used

From η the present baryon density can be found as

$$\Omega_b h^2 = 0.0037\, \eta_{10}$$

Immediately after the quark-hadron transition almost all baryons are in pions. However, when the temperature has dropped to a few MeV (T << m_π) only neutrons and protons are left

In thermal equilibrium

$$\frac{n_n}{n_p} = \exp(-\Delta m / T) \quad , \quad \Delta m = 1.293\,\mathrm{MeV}$$

However, this ratio is dependent on weak interaction equilibrium *n-p* changing reactions

$$\nu_e + n \leftrightarrow e^- + p$$
$$e^+ + n \leftrightarrow \bar{\nu}_e + p$$
$$n \leftrightarrow e^- + p + \bar{\nu}_e$$

Interaction rate (the generic weak interaction rate)

$$\Gamma_{n-p} = n\langle \sigma v \rangle \approx T^3 G_F^2 T^2 \Rightarrow T_{freeze} \approx 1\,\mathrm{MeV}$$

After that, neutrons decay freely with a lifetime of

$$\tau_n = 886 \pm 0.8\ s$$

However, before complete decay neutrons are bound in nuclei.

Nucleosynthesis should intuitively start when
$T \sim E_b$ (D) ~ 2.2 MeV via the reaction

$$\boxed{p + n \leftrightarrow D + \gamma}$$

However, because of the high entropy it does not.
Instead the nucleosynthesis starting point can be found from the condition $\Gamma_{production}(D) = \Gamma_{destruction}(D)$

$$\left.\begin{aligned} \Gamma_{production} &\approx n_B \langle \sigma v \rangle \\ \Gamma_{destruction} &\approx n_\gamma \langle \sigma v \rangle e^{-E_b/T} \end{aligned}\right\} \Rightarrow T_{BBN} \approx -\frac{E_b}{\ln(\eta)} \approx 0.2\,\mathrm{MeV}$$

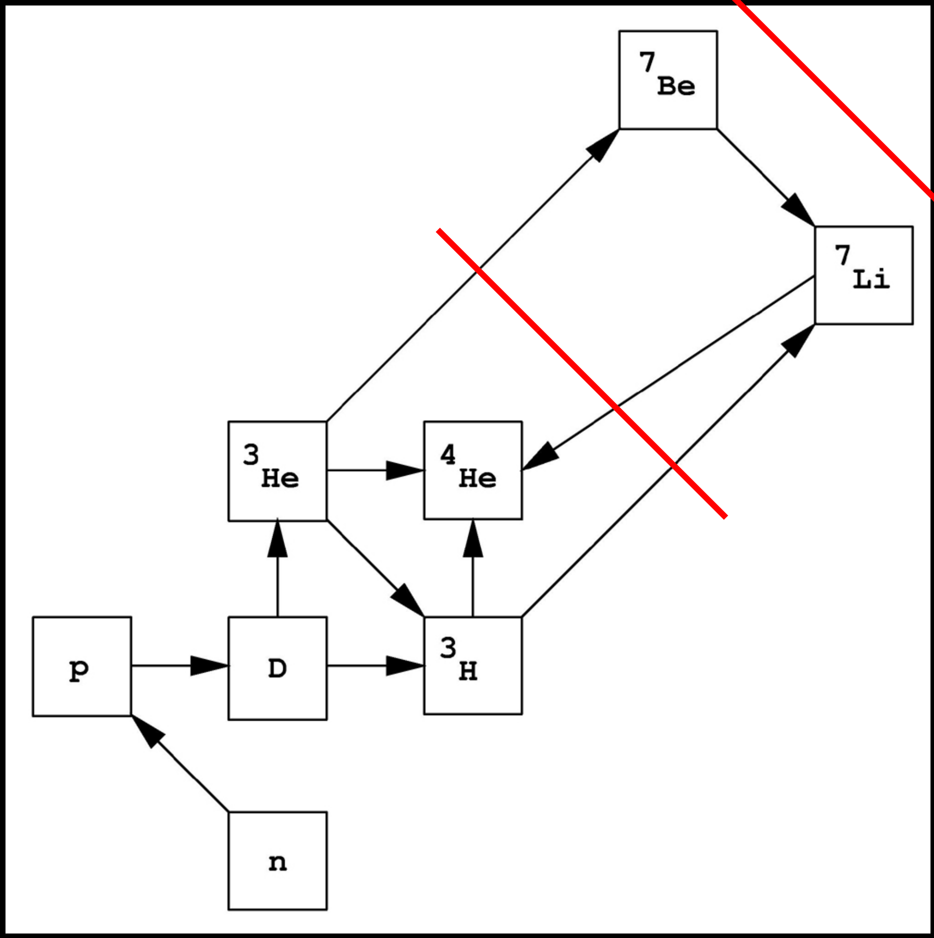
Since $t(T_{\mathrm{BBN}}) \sim 50$ s << τ_{n} only few neutrons have time to decay

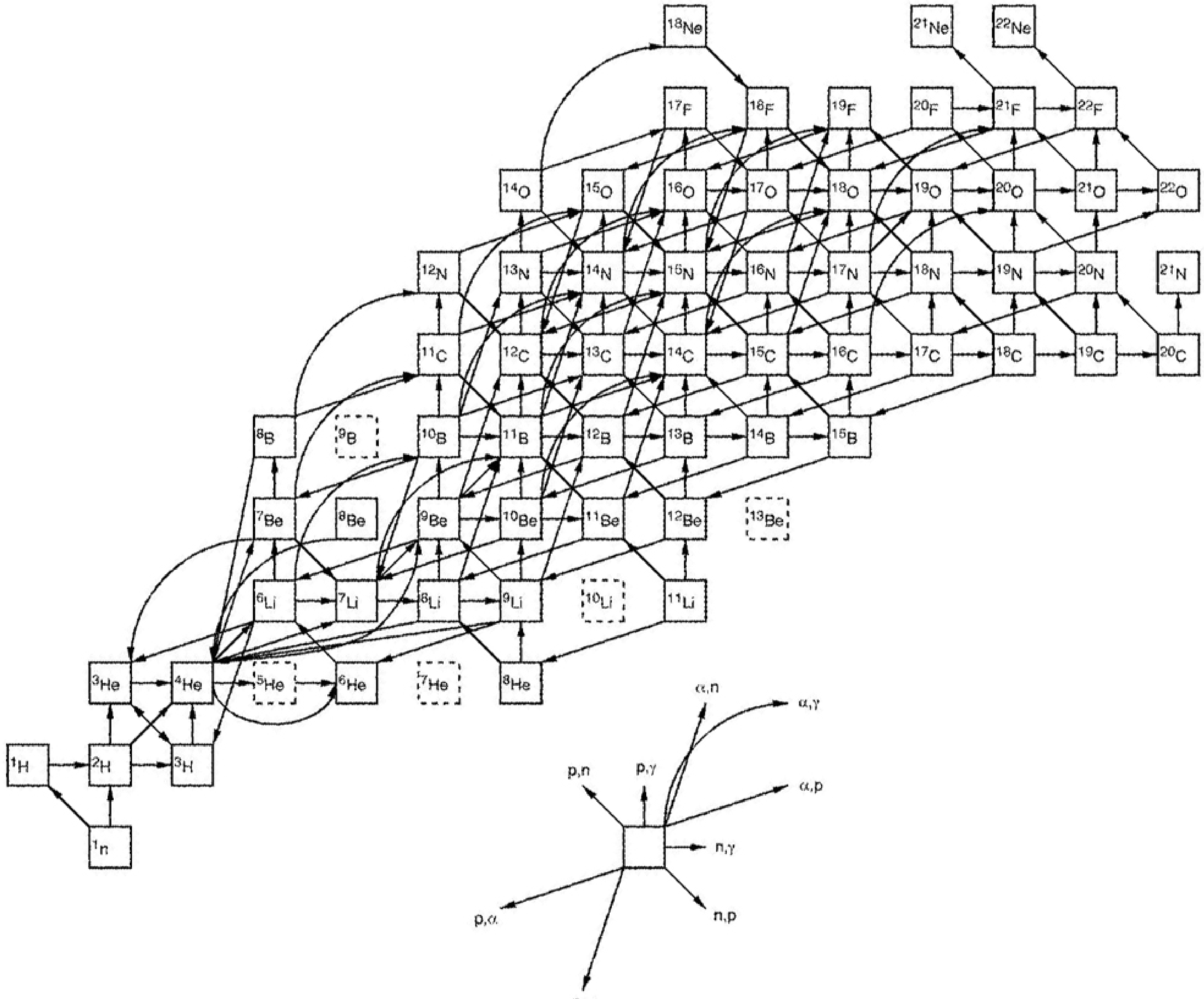
At this temperature nucleosynthesis proceeds via the reaction network

The mass gaps at A = 5 and 8 lead to small production of mass numbers 6 and 7, and almost no production of mass numbers above 8

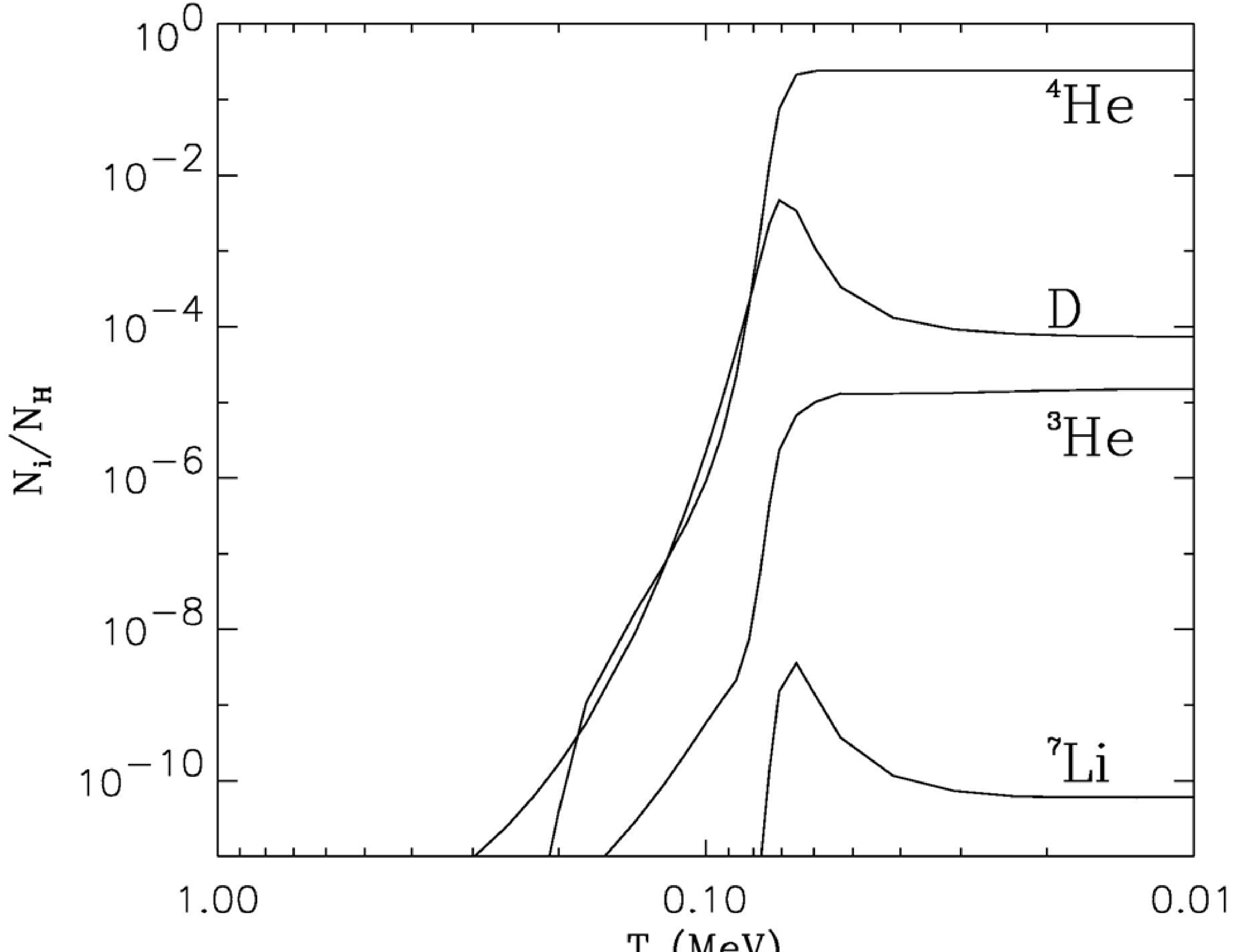
The gap at A = 5 can be spanned by the reactions

$$^{3}He(^{4}He,\gamma)^{7}Be$$
$$T(^{4}He,\gamma)^{7}Li$$



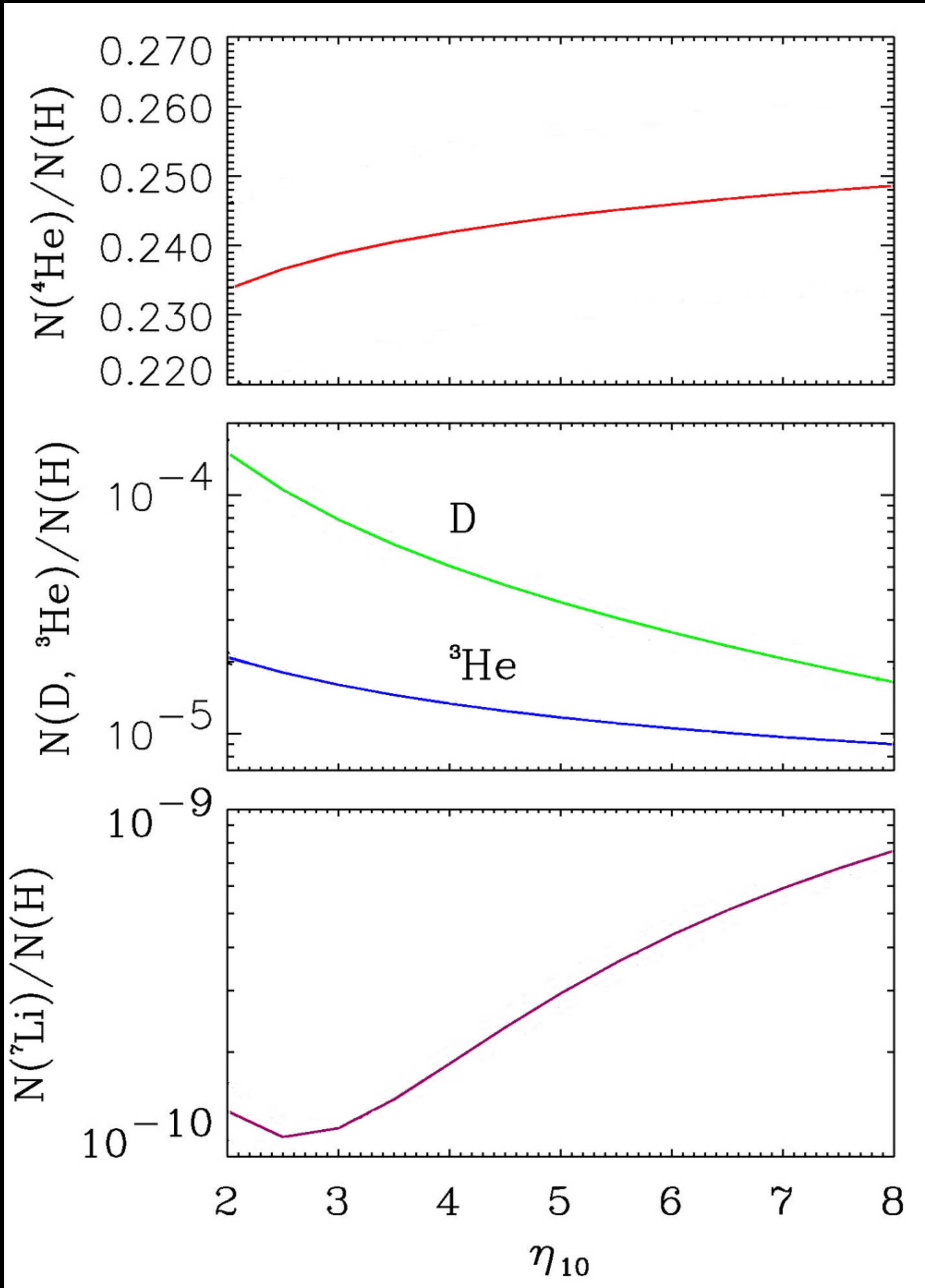


18Ne
21Ne
22Ne
17F
18F
19F
20F
21F
22F
14O
15O
16O
17O
18O
19O
20O
21O
22O
12N
13N
14N
15N
16N
17N
18N
19N
20N
21N
11C
12C
13C
14C
15C
16C
17C
18C
19C
20C
8B
9B
10B
11B
12B
13B
14B
15B
7Be
8Be
9Be
10Be
11Be
12Be
13Be
6Li
7Li
8Li
9Li
10Li
11Li
3He
4He
5He
6He
7He
8He
1H
2H
3H
1n
α,n
α,γ
p,n
p,γ
α,p
n,γ
p,α
n,p
n,α



10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
N_i/N_H
^{4}He
D
^{3}He
^{7}Li
1.00
0.10
0.01
T (MeV)

The amounts of various elements produced depend on the physical conditions during nucleosynthesis, primarily the values of *N*(T) and η



Helium-4: Essentially all available neutrons are processed into He-4, giving a mass fraction of

$$Y_P = \frac{4n_{He}}{n_N} = \left.\frac{2n_n}{n_n + n_p}\right|_{T_{BBN}} \approx 0.25 \quad \text{for} \quad n_n / n_p \sim 1/7$$

$$\left.\frac{n_n}{n_p}\right|_{T_{BBN}} \approx \exp(-\Delta m / T_{weak}) \frac{\exp(-t_{BBN} / \tau_n)}{2 - \exp(-t_{BBN} / \tau_n)} \sim 1/7$$

Y_p depends on η because T_{BBN} changes with η

$$T_{BBN} \approx -\frac{E_{B,D}}{\ln(\eta)}$$

D, He-3: These elements are processed to produce He-4. For higher η, T_{BBN} is higher and they are processed more efficiently

Li-7: Non-monotonic dependence because of two different production processes

Much lower abundance because of mass gap

Confronting theory with observations

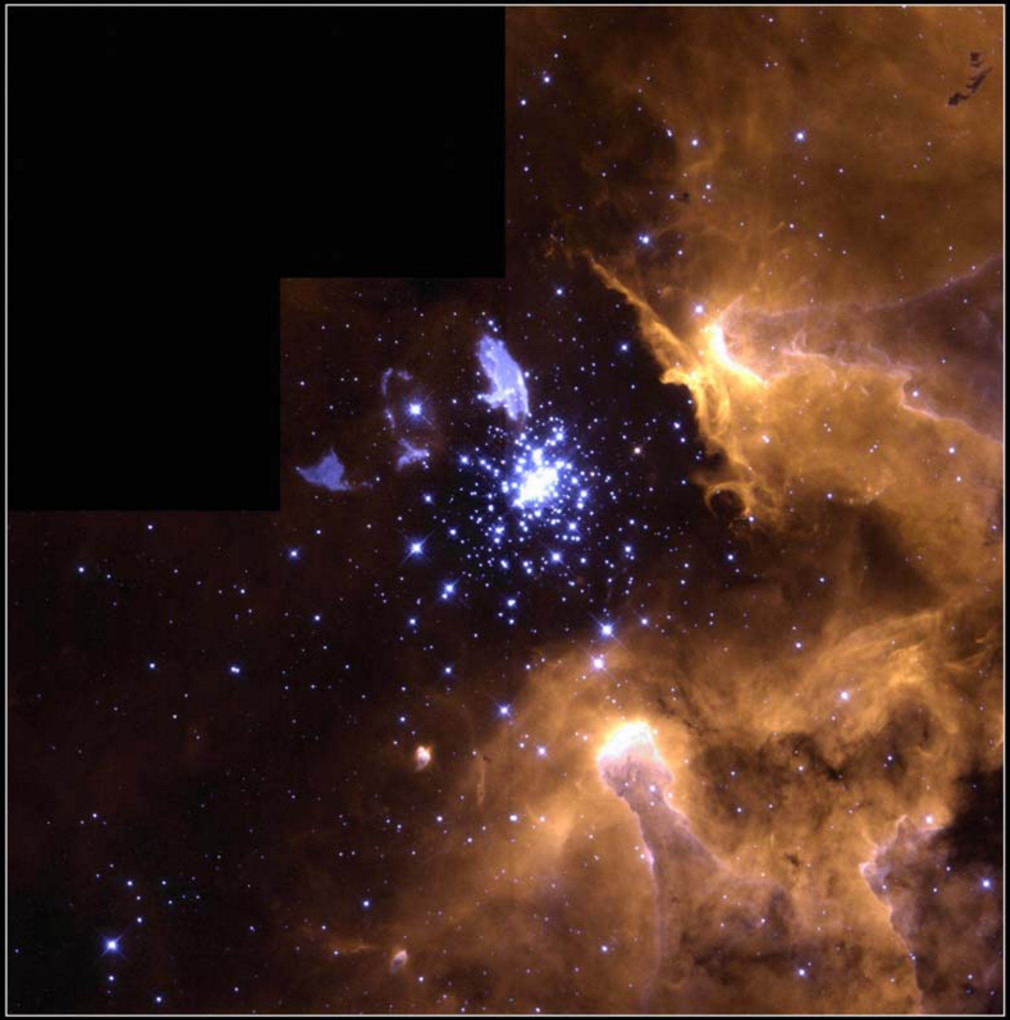
He-4:

He-4 is extremely stable and is in general always produced, not destroyed, in astrophysical environments

The Solar abundance is $Y = 0.28$, but this is processed material

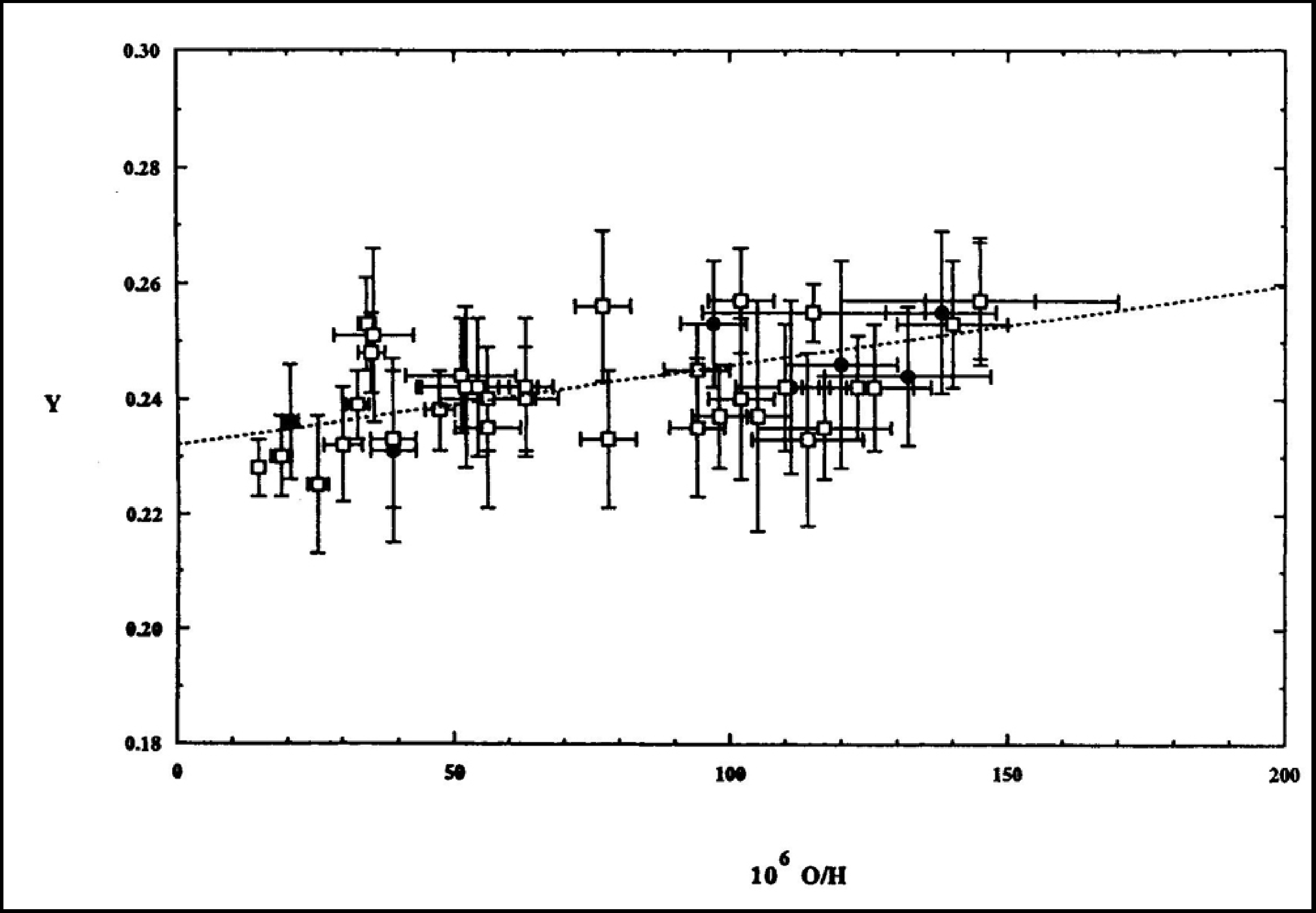
The primordial value can in principle be found by measuring He abundance in unprocessed (low metallicity) material.

Extragalactic H-II regions

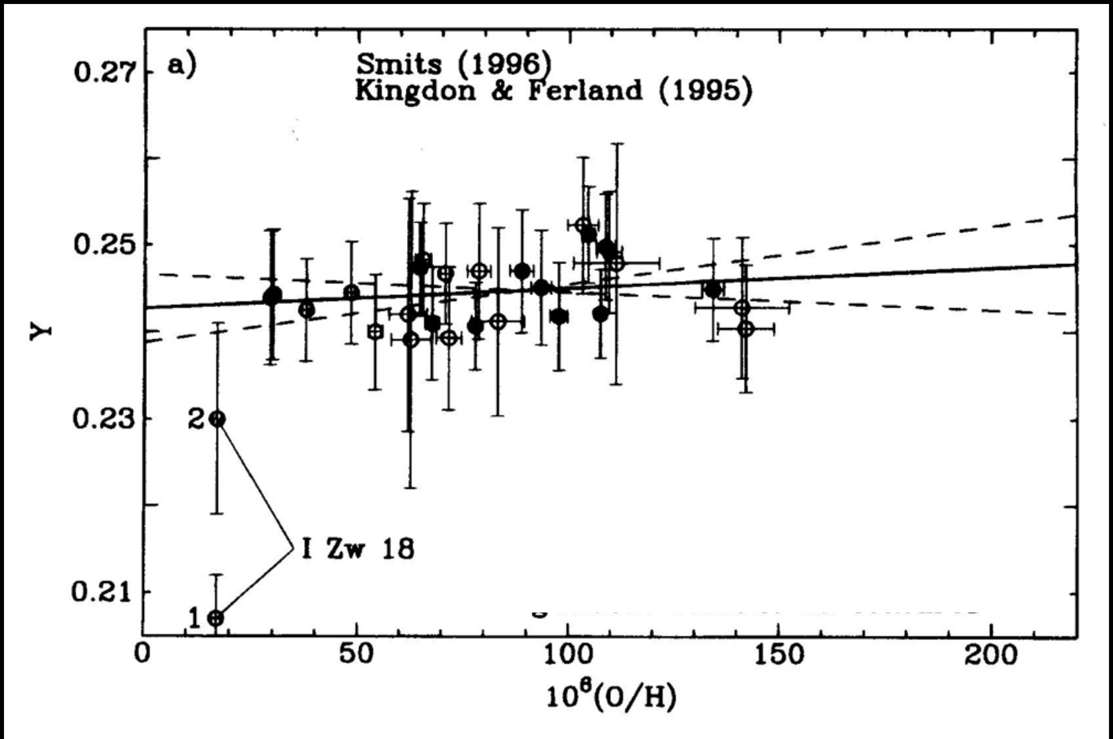


NGC 3603
Hubble Space Telescope • WFPC2

PRC99-20 • STScI OPO
Wolfgang Brandner (JPL/IPAC), Eva K. Grebel (University of Washington),
You-Hua Chu (University of Illinois, Urbana-Champaign) and NASA



Olive, Skillman & Steigman



Izotov & Thuan

Most recent values:

Fields & Olive : $Y = 0.238 \pm 0.002 \pm 0.005$

Izotov & Thuan : $Y = 0.244 \pm 0.002$

Deuterium: Deuterium is weakly bound and therefore can be assumed to be only destroyed in astrophysical environments

Primordial deuterium can be found either by measuring solar system or ISM value and doing complex chemical evolution calculations

OR

Measuring D at high redshift

The ISM value of

$$(D/H)_{ISM} = 1.60 \pm 0.09^{+0.05}_{-0.10} \times 10^{-5}$$

can be regarded as a firm lower bound on primordial D

1994: First measurements of D in high-redshift absorption systems

A very high D/H value was found

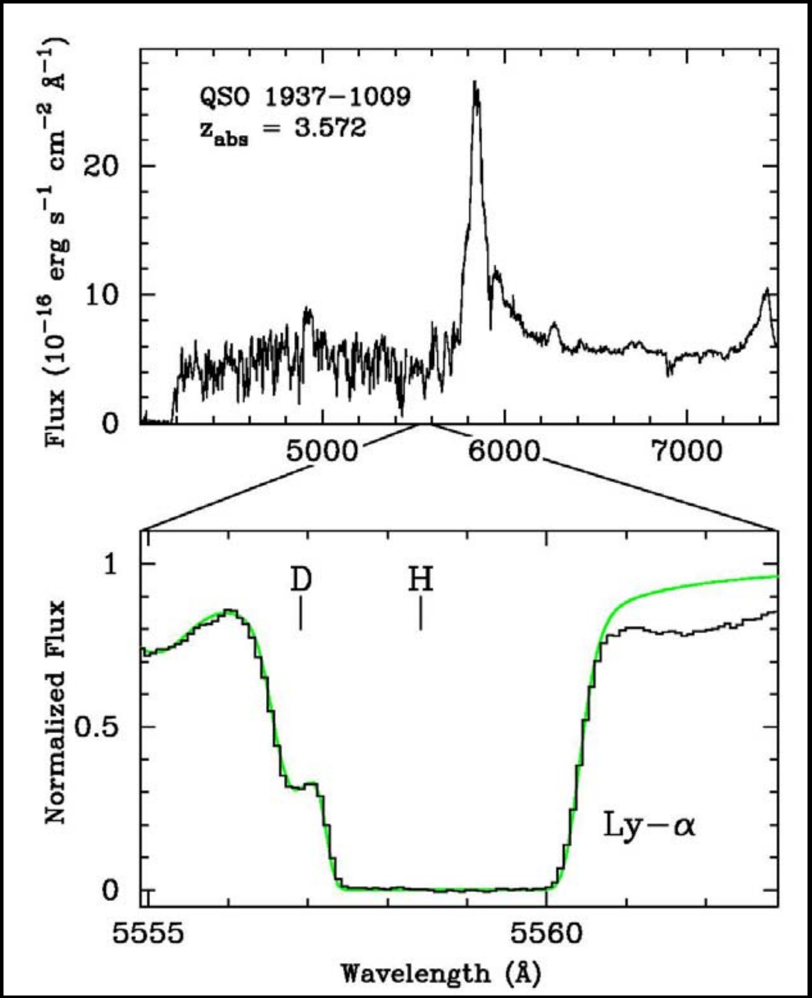
$$(D/H)_{High-z} \approx 1.9 - 2.5 \times 10^{-4}$$

Carswell et al. 1994
Songaila et al. 1994

However, other measurements found much lower values

$$(D/H)_{High-z} \approx 2.5 \times 10^{-5}$$

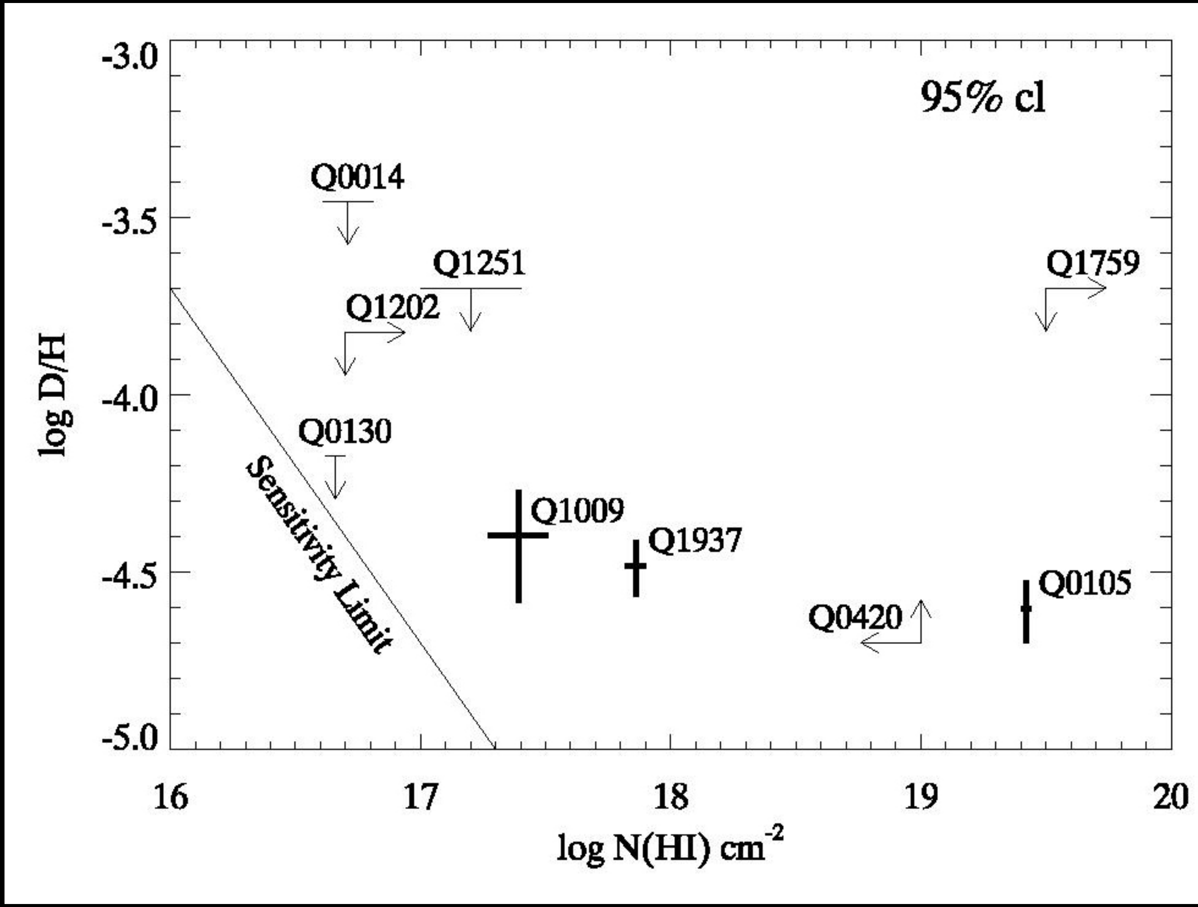
Burles & Tytler 1996



Burles & Tytler

The discrepancy has been "resolved" in favour of a low deuterium value of roughly

$$(D/H)_{High-z} \approx 3.4 \pm 0.5 \times 10^{-5}$$



Burles, Nollett & Turner 2001

Li-7: Lithium can be both produced and destroyed in astrophysical environments

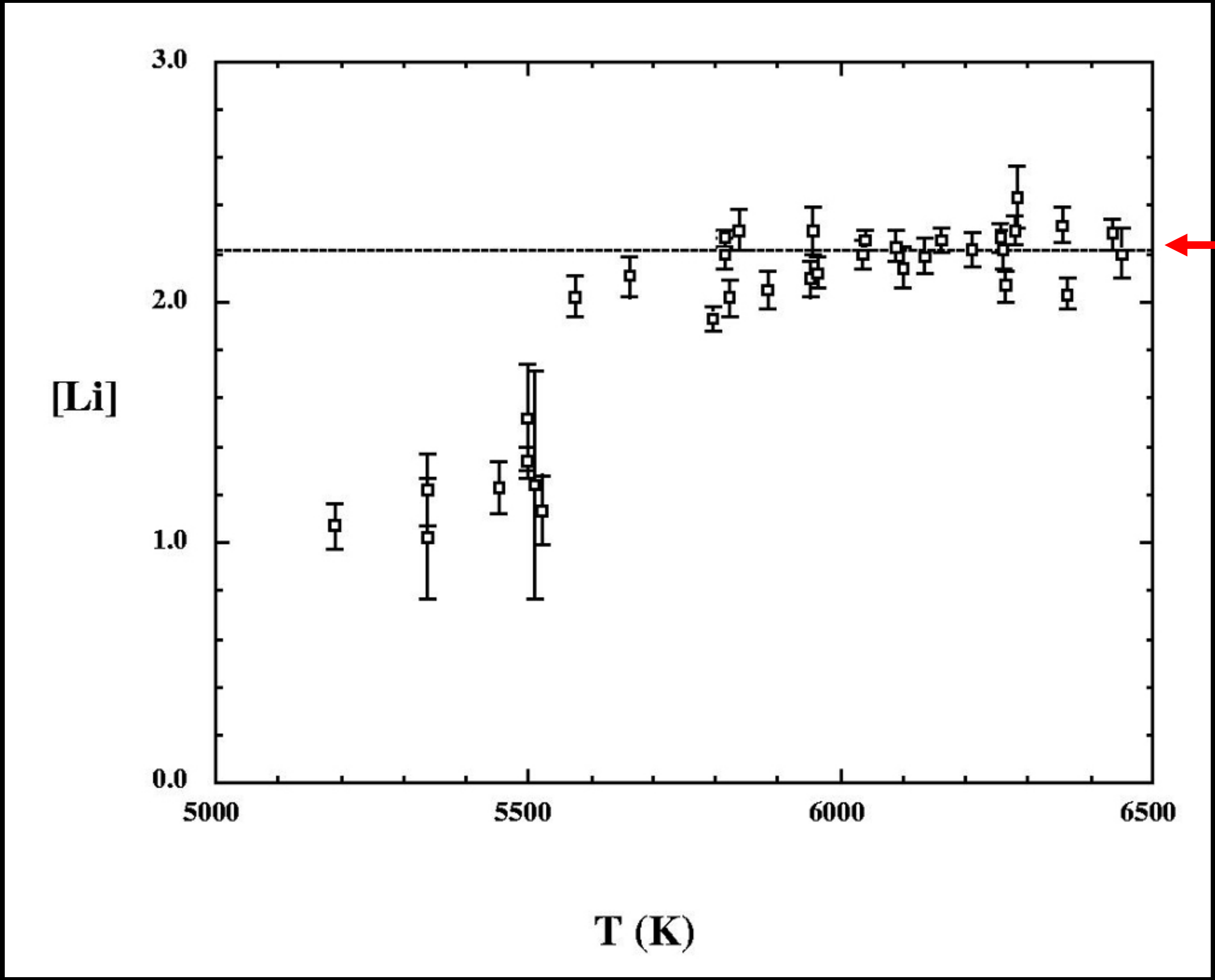
Production is mainly by cosmic ray interactions

Destruction is in stellar interiors

Old, hot halo stars seem to be good probes of the primordial Li abundance because there has been only limited Li destruction

Li-abundance in old halo stars in units of

$$[\mathrm{Li}] = \log(^{7}\mathrm{Li}/\mathrm{H}) + 12$$



Spite plateau

Molaro et al. 1995

There is consistency between theory and observations

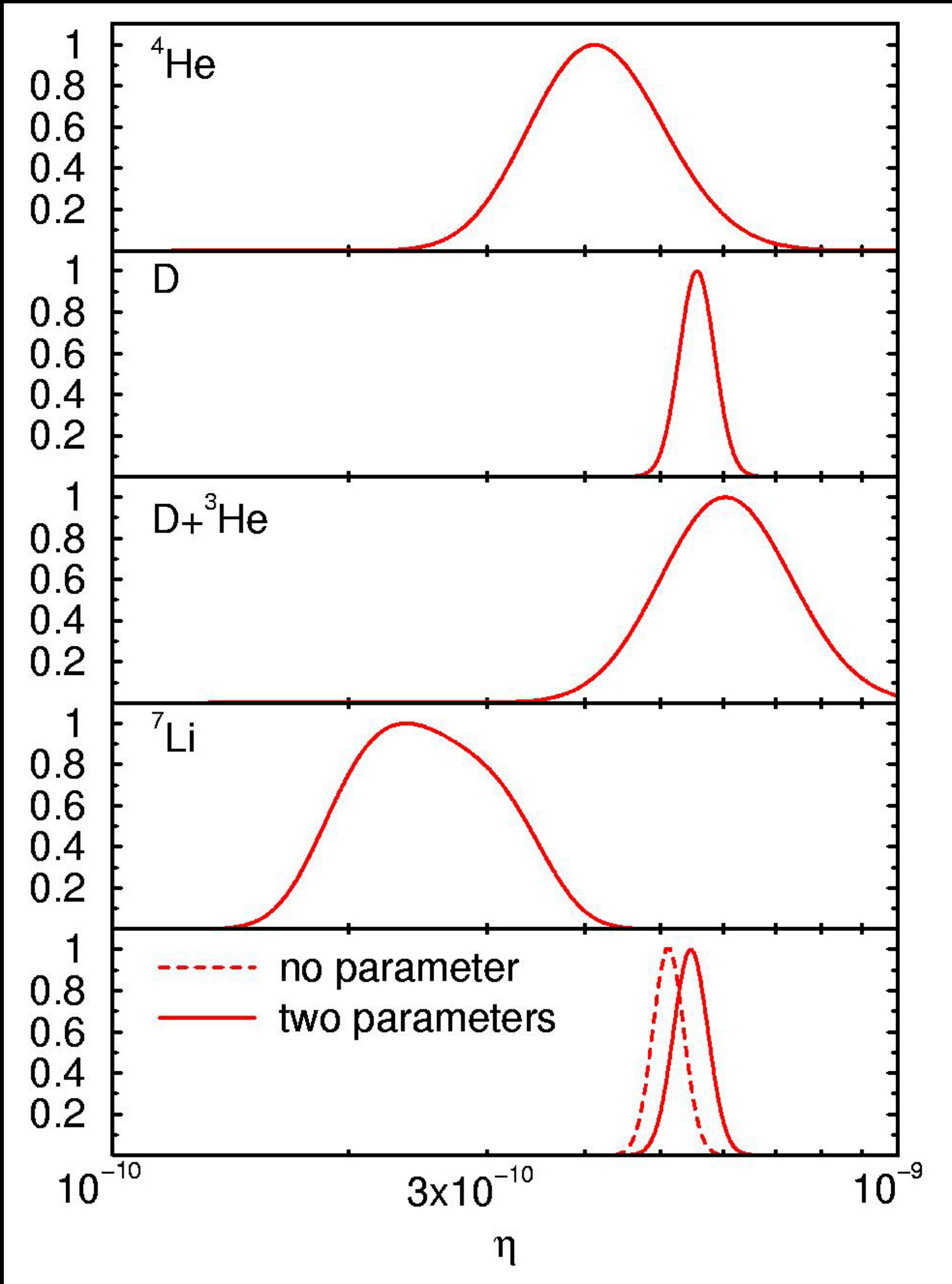
All observed abundances fit well with a single value of eta

This value is mainly determined by the High-z deuterium measurements

The overall best fit is

$$\eta = 5.1 \pm 0.3 \times 10^{-10}$$

Burles, Nollett & Turner 2001



This value of η translates into

$$\Omega_b h^2 = 0.020 \pm 0.002$$

And from the HST value for *h*

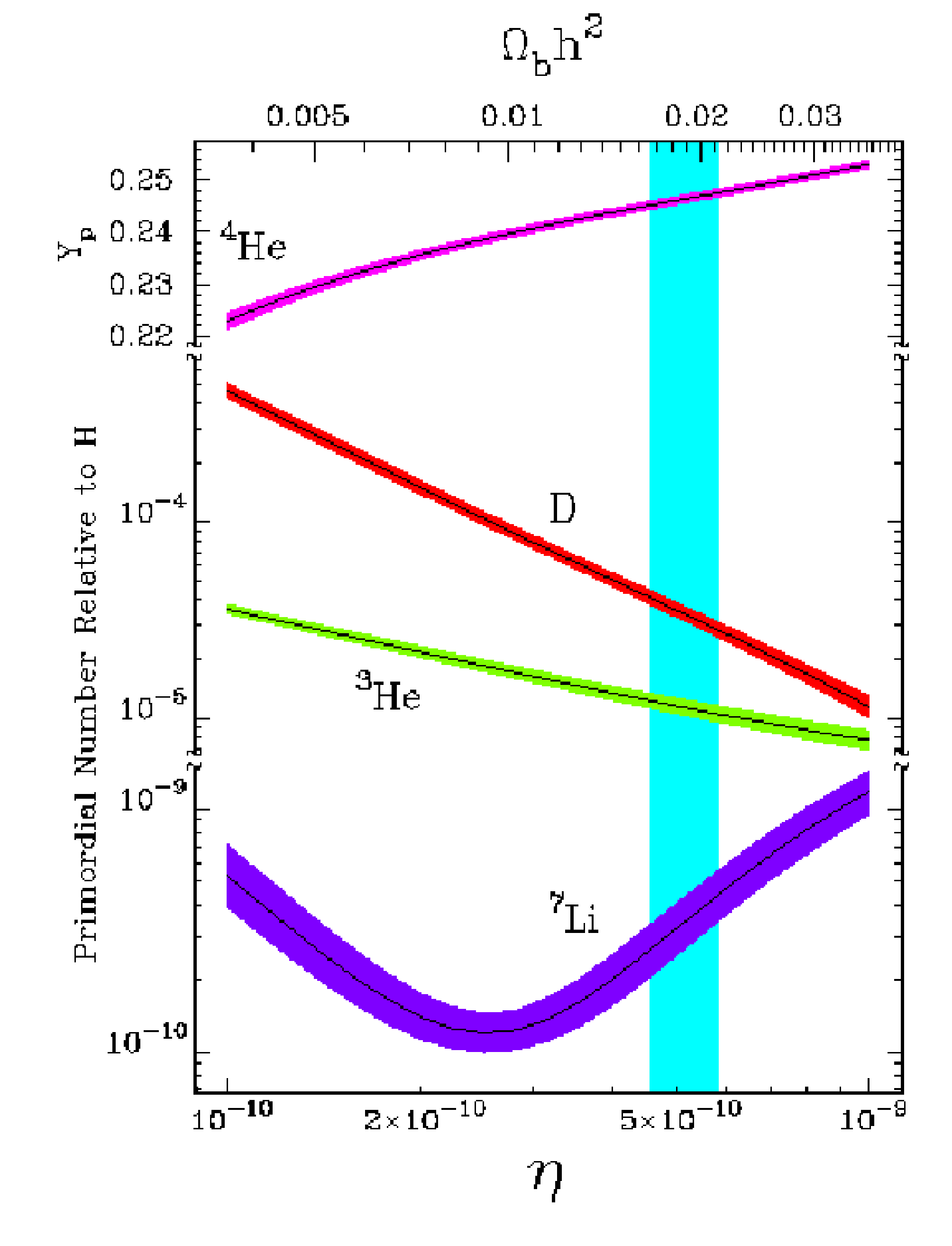
$$h = 0.72 \pm 0.08$$

One finds

$$0.028 \le \Omega_b \le 0.054$$

$$\Omega_{\text{luminous}} \le 0.01$$

$$\Omega_m \approx 0.3$$



BOUND ON THE RELATIVISTIC ENERGY DENSITY (NUMBER OF NEUTRINO SPECIES) FROM BBN

The weak decoupling temperature depends on the expansion rate

$$H = \sqrt{\frac{8\pi G\rho}{3}} = \sqrt{\frac{2\pi^3 G N(T) T^4}{15}}$$

And decoupling occurs when

$$\Gamma_{\text{int}} \approx G_F^2 T^5 \approx H \Rightarrow T_D \propto N(T)^{1/6}$$

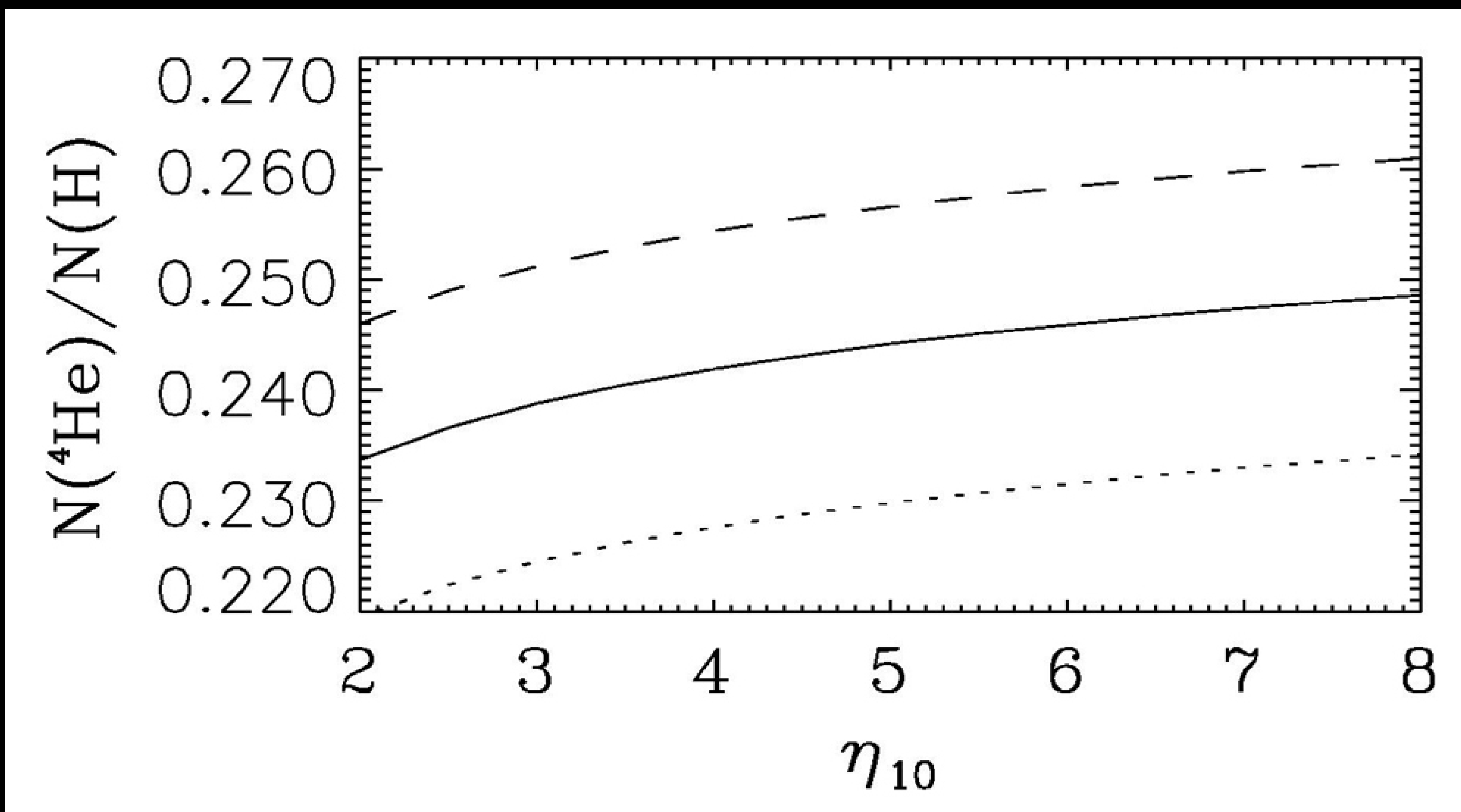
N(*T*) is can be written as

$$\begin{aligned} N(T) &= N(T)_{e^+,e^-,\gamma} + N(T)_{\nu,SM} \frac{\rho_{\nu+\text{extra}}}{\rho_{\nu,SM}} \\ &= N(T)_{e^+,e^-,\gamma} + N(T)_{\nu,SM} (3 + \Delta N_\nu) \end{aligned}$$

Since

$$\left.\frac{n_n}{n_p}\right|_{BBN} \approx \exp(-\Delta m / T_D)$$

The helium production is very sensitive to N_ν



$N_\nu = 4$

$N_\nu = 3$

$N_\nu = 2$

Using BBN to probe physics beyond the standard model

Non-standard physics can in general affect either

Expansion rate during BBN

- extra relativistic species
- massive decaying particles
- quintessence
- ….

The interaction rates themselves

- neutrino degeneracy
- changing fine structure constant
- ….

Example: A changing fine-structure constant α

Many theories with extra dimensions predict 4D-coupling constants which are functions of the extra-dimensional space volume.

Webb et al.: Report evidence for a change in α at the $\Delta\alpha/\alpha \sim 10^{-5}$ level in quasars at $z \sim 3$

BBN constraint:

BBN is useful because a changing α would change all EM interaction rates and change nuclear abundance

Bergström et al: $\Delta\alpha/\alpha < 0.05$ at $z \sim 10^{12}$