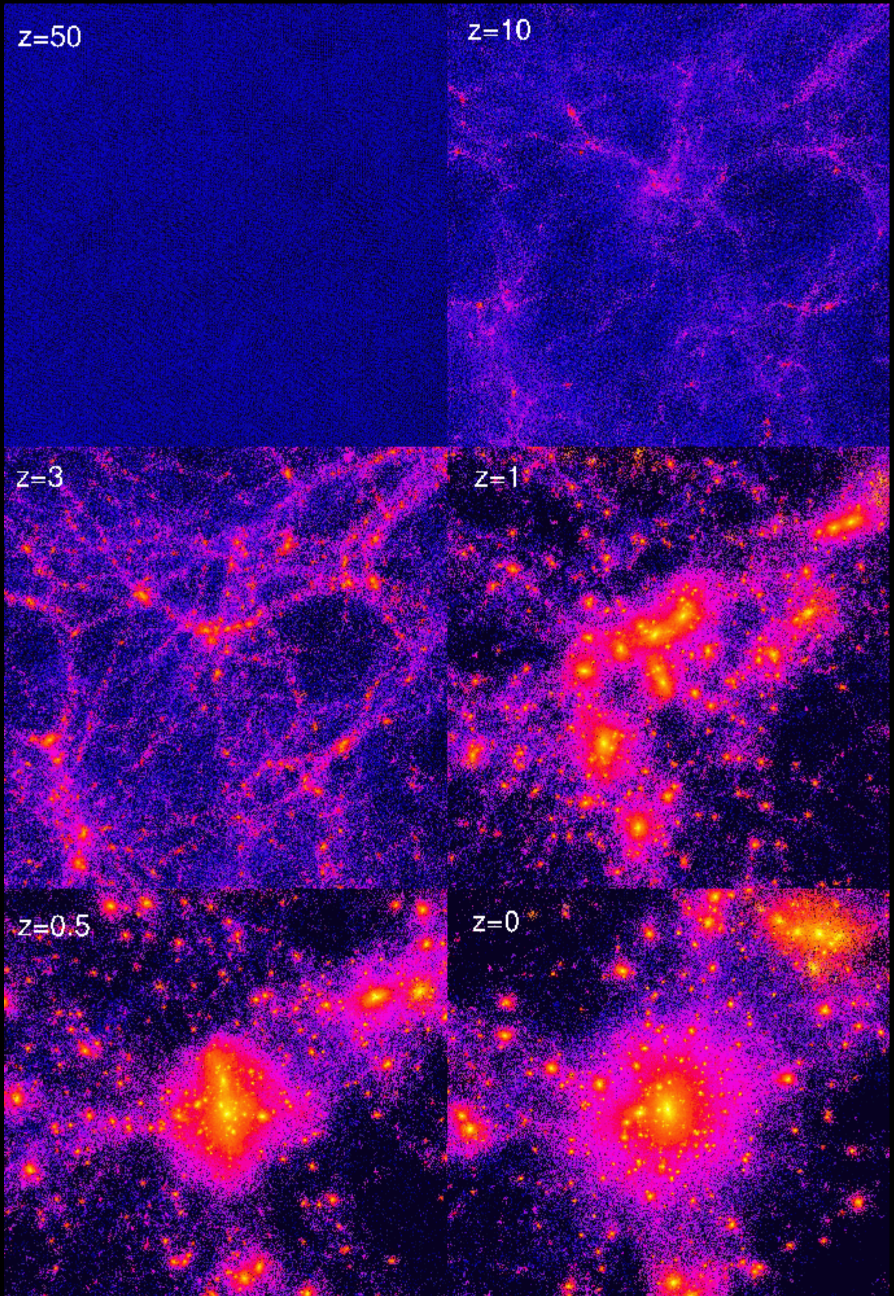
STRUCTURE FORMATION IN THE UNIVERSE



GROWTH OF PERTURBATIONS, THE OLD JEANS ANALYSIS (APPLIES ALSO TO STAR FORMATION)

CONTINUITY: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

EULER: $$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} + \frac{\nabla P}{\rho} + \nabla \phi = 0$$

POISSON: $$\nabla^2 \phi = 4\pi G \rho$$

THESE EQUATIONS CAN BE LINEARIZED

$$\rho = \rho_0 + \rho_1$$

$$P = P_0 + P_1$$

$$\vec{v} = \vec{v}_0 + \vec{v}_1$$

$$\phi = \phi_0 + \phi_1$$

FROM THIS ASSUMPTION THE FOLLOWING EQUATIONS ARE DERIVED FOR THE FIRST ORDER TERMS

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \vec{v}_1 = 0$$

$$\frac{\partial \vec{v}_1}{\partial t} + v_s^2 \frac{\nabla \rho_1}{\rho_0} + \nabla \phi_1 = 0$$

$$\nabla^2 \phi_1 = 4\pi G \rho_1$$

FOR ρ THIS CAN BE COMBINED INTO A SINGLE SECOND ORDER DIFFERENTIAL EQUATION

$$\frac{\partial^2 \rho_1}{\partial t^2} - v_s^2 \nabla^2 \rho_1 = 4\pi G \rho_0 \rho_1$$

THIS IS RECOGNISEABLE AS A SIMPLE HARMONIC OSCILLATOR EQUATION (WAVE EQUATION),
I.E. THE SOLUTIONS ARE

$$\rho_1 = Ke^{-i(\vec{k}\cdot\vec{r}+\omega t)}$$

$$\omega^2 = v_s^2 k^2 - 4\pi G\rho_0 \begin{cases} \omega^2 < 0 \Rightarrow \text{exponential growth} \\ \omega^2 > 0 \Rightarrow \text{oscillations} \end{cases}$$

DIVIDING WAVENUMBER (INVERSE LENGTH SCALE) IS CALLED THE JEANS' WAVENUMBER

$$k_J = \left(\frac{4\pi G\rho_0}{v_s^2}\right)^{1/2}$$

THE NATURE OF MODES ON A GIVEN SCALE IS SIMPLY DETERMINED FROM THE RELATIVE STRENGTH OF GRAVITATIONAL AND PRESSURE FORCES

NOW, WHAT HAPPENS IF THE UNIVERSE IS EXPANDING?

EXACTLY THE SAME ANALYSIS CAN BE PERFORMED, WITH THE ONE EXCEPTION THAT THE UNPERTURBED SOLUTIONS ARE THEN

$$\rho_0(t) = \rho_0(t) a^{-3}$$

$$\vec{v}_0 = \frac{\dot{a}}{a}\vec{r}$$

$$\nabla \phi_0 = \frac{4\pi G \rho_0}{3}\vec{r}$$

THE CORRESPONDING EQUATIONS FOR THE FIRST ORDER QUANTITIES ARE THEN

$$\frac{\partial \rho_1}{\partial t} + 3\frac{\dot{a}}{a}\rho_1 + \frac{\dot{a}}{a}(\vec{r}\cdot\nabla)\rho_1 + \rho_0 \nabla\cdot\vec{v}_1 = 0$$

$$\frac{\partial \vec{v}_1}{\partial t} + \frac{\dot{a}}{a}\vec{v}_1 + \frac{\dot{a}}{a}(\vec{r}\cdot\nabla)\vec{v}_1 + v_s^2\frac{\nabla\rho_1}{\rho_0} + \nabla\phi_1 = 0$$

$$\nabla^2\phi_1 = 4\pi G\rho_1$$

IT TURNS OUT THAT IN FOURIER SPACE THESE EQUATIONS ARE MUCH SIMPLER, SO ALL QUANTITIES SHOULD BE EXPANDED IN FOURIER MODES

$$\psi(\vec{r},t) = \frac{1}{(2\pi)^3}\int \psi_k(t) e^{-i\vec{k}\cdot\vec{r}} d^3 r$$

THE EQUATION FOR THE DENSITY PERTURBATION THEN BECOMES

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + \left(\frac{v_s^2 k^2}{a^2} - 4\pi G\rho_0\right)\delta_k = 0$$

THIS IS EXACTLY THE SAME EQUATION AS IN THE NON-EXPANDING CASE, EXCEPT FOR THE SECOND TERM

THE EXPANSION OF THE UNIVERSE ACTS LIKE A FRICTION FORCE

EVEN IF THERE ARE GROWING MODES, THEY CANNOT BE EXPONENTIAL

IN A FLAT MODEL WITH ONLY CDM THE EQUATION BECOMES

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

WITH SOLUTIONS

$$\delta \sim t^{2/3} \quad \wedge \quad \delta \sim t^{-1}$$

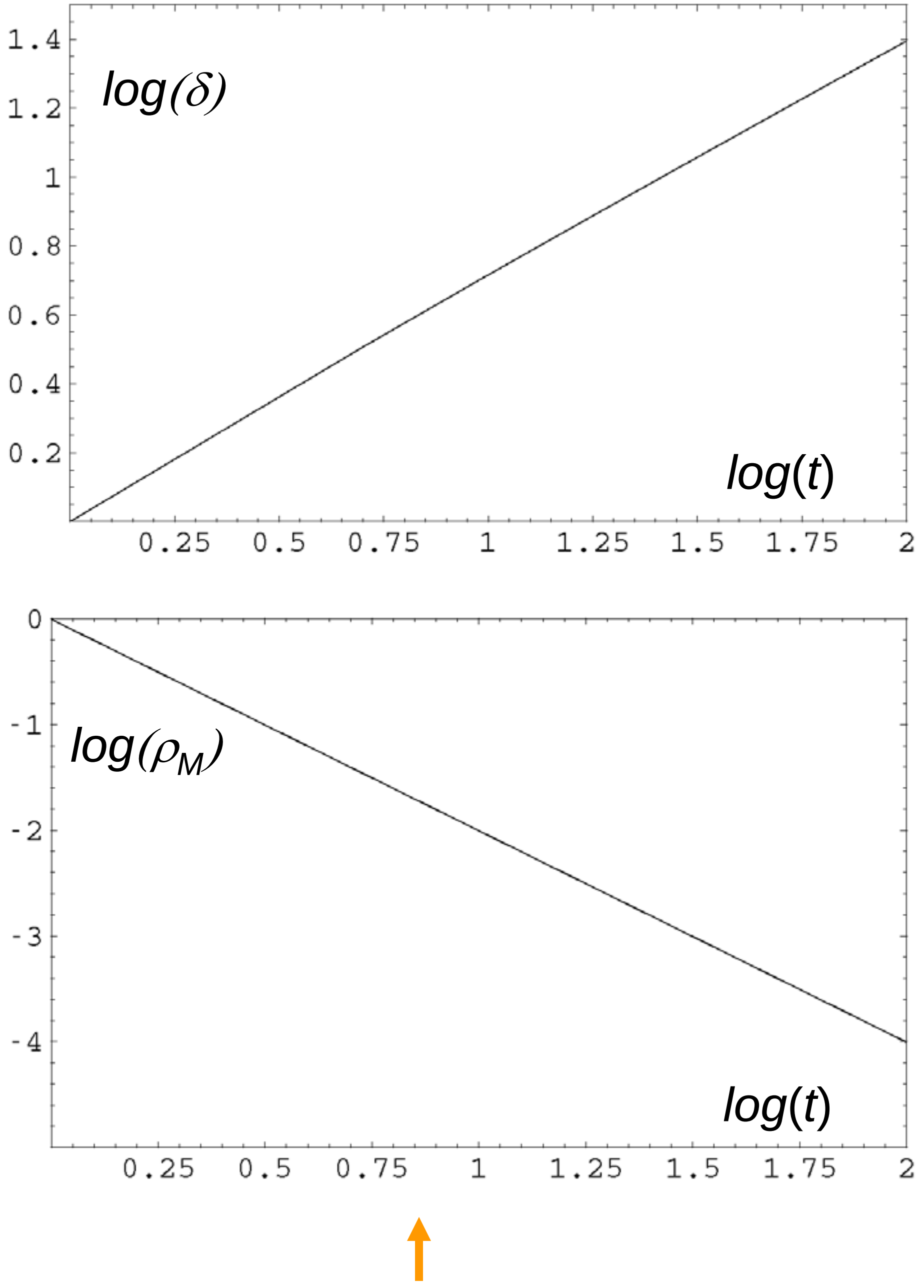
FOR A MODEL DOMINATED BY Λ THE SOLUTIONS ARE

$$\delta \sim K \quad \wedge \quad \delta \sim e^{-2Ht}$$

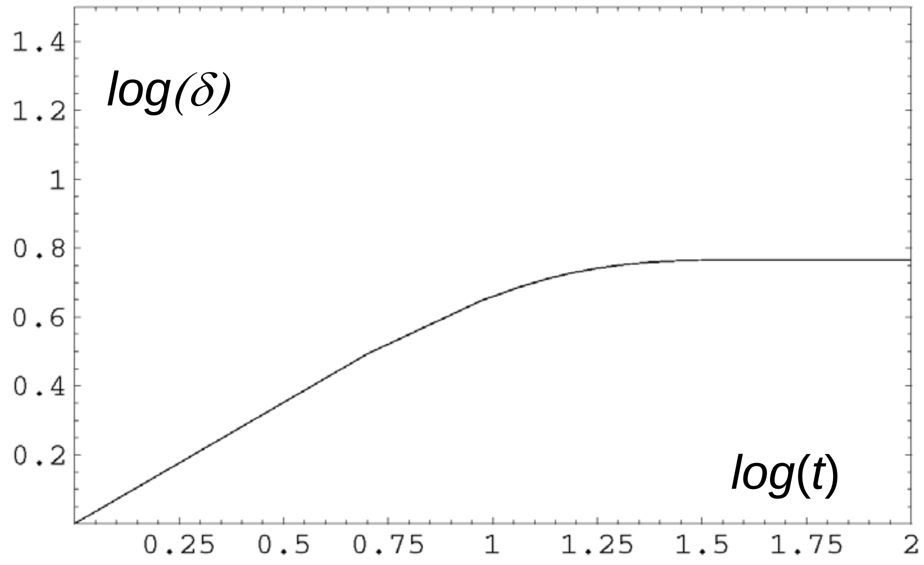
AND FOR A CURVATURE DOMINATED MODEL

$$\delta \sim K \quad \wedge \quad \delta \sim t^{-1}$$

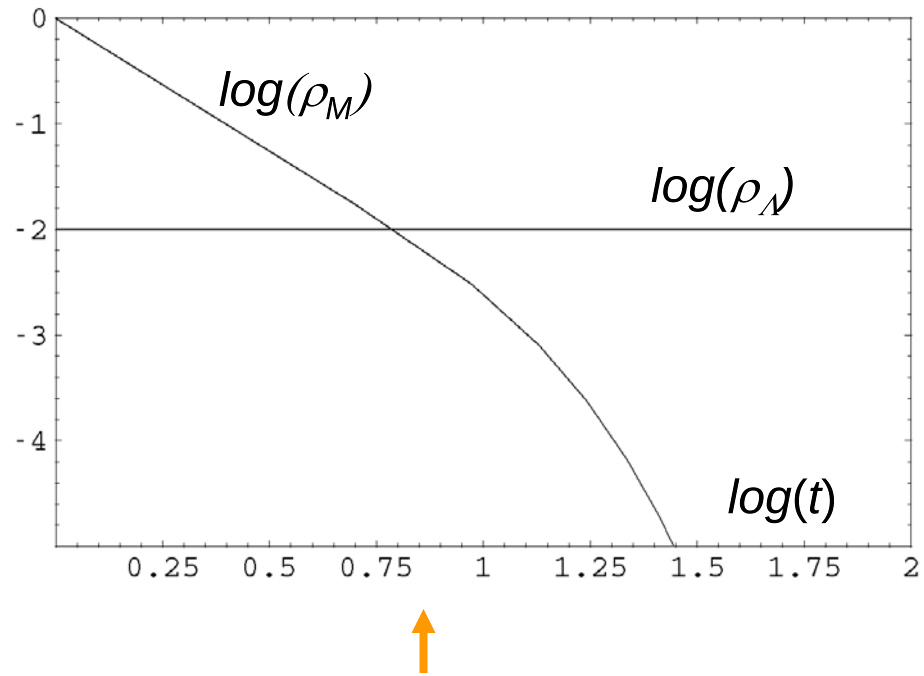
ONLY THE MATTER DOMINATED EPOCH ALLOWS FOR POWER-LAW GROWING MODES!



$log(\delta)$
1.4
1.2
1
0.8
0.6
0.4
0.2
$log(t)$
0.25
0.5
0.75
1
1.25
1.5
1.75
2
$log(\rho_M)$
0
-1
-2
-3
-4
$log(t)$
0.25
0.5
0.75
1
1.25
1.5
1.75
2



1.4
1.2
1
0.8
0.6
0.4
0.2
$log(\delta)$
$log(t)$
0.25
0.5
0.75
1
1.25
1.5
1.75
2



0
-1
-2
-3
-4
$log(\rho_M)$
$log(\rho_\Lambda)$
$log(t)$
0.25
0.5
0.75
1
1.25
1.5
1.75
2

THE R.M.S. AMOUNT OF DENSITY FLUCTUATIONS IN THE UNIVERSE CAN BE WRITTEN AS

$$\frac{\delta\rho}{\rho} = \left\langle \delta(\vec{x})\delta(\vec{x}) \right\rangle^{1/2}$$

THIS FLUCTUATION CAN BE DECOMPOSED NATURALLY IN FOURIER MODES

$$\frac{\delta\rho}{\rho} = K \int k^2 \left|\delta_k\right|^2 dk$$

WHERE $\left|\delta_k\right|^2$ IS CALLED THE POWER SPECTRUM (NORMALLY CALLED *P*(*k*))

IT IS THE FOURIER TRANSFORM OF THE TWO-POINT CORRELATION FUNCTION

THE POWER SPECTRUM CAN BE DECOMPOSED INTO AN INITIAL, PRIMORDIAL POWER SPECTRUM, AND A TRANSFER FUNCTION, RELATED TO THE TIME-EVOLUTION OF PERTURBATIONS VIA THE PREVIOUSLY DERIVED EQUATIONS

$$P(k,t) = T^2(k,t)P_i(k)$$

$P_i(k)$ IS NORMALLY (WITH GOOD REASON) PARAMETRIZED AS A SIMPLE POWER-LAW

$$P_i(k) = Ak^{n-1}$$

WHERE n IS CALLED THE SPECTRAL INDEX.

IN GENERAL RELATIVITY THE FORM OF THE EQUATIONS AT SMALL SCALES IS THE SAME AS IN THE NEWTONIAN DERIVATION

HOWEVER, ON SCALES CLOSE OR, OR ABOVE THE CAUSAL HORIZON THAT IS NO LONGER TRUE

INSTEAD ONE WORKS FROM A PERTURBATION AROUND THE FRW METRIC

$$ds^2 = dt^2 - a^2(t)[(\delta_{ij} + h_{ij})dx_i dx_j]$$

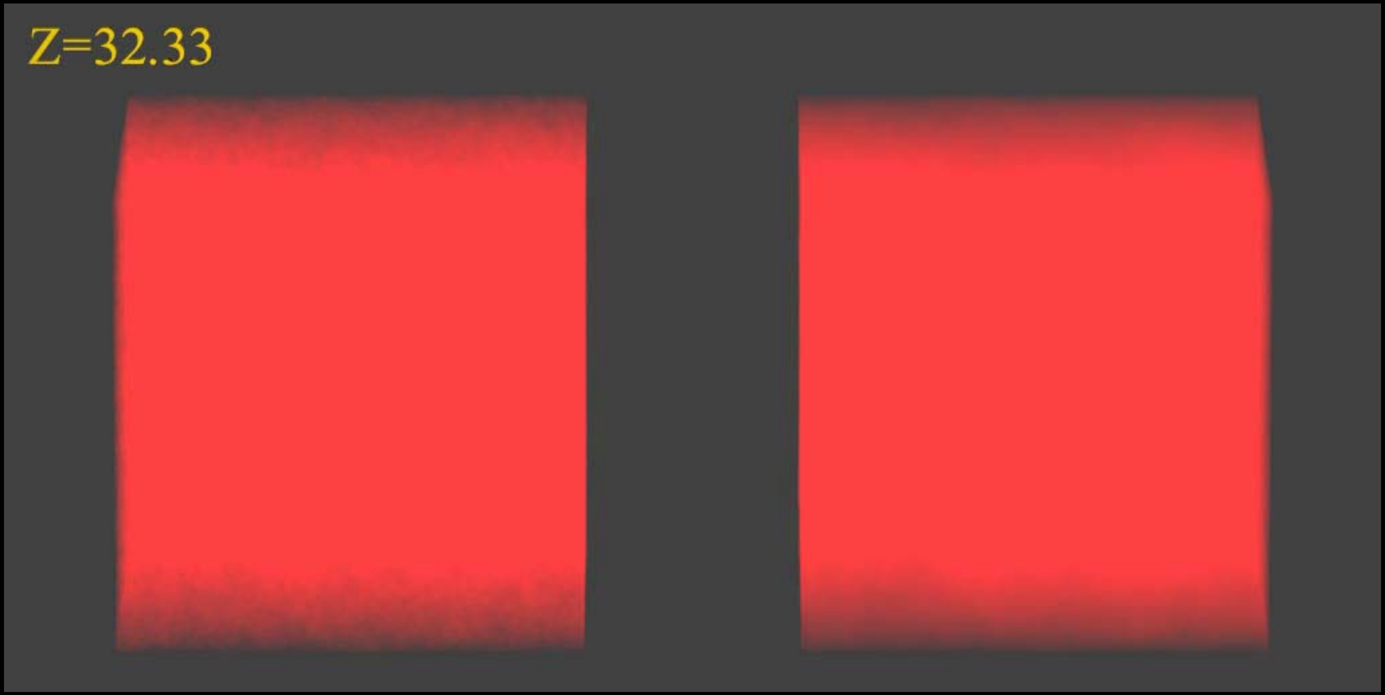
δ_{ij} is the unperturbed FRW part

h_{ij} is the perturbation

THE MAIN DIFFERENCE IS THAT THE GENERAL RELATIVISTIC PERTURBATIONS CAN HAVE TENSOR CHARACTER, I.E. BE GRAVITATIONAL WAVES

N-BODY SIMULATIONS OF ΛCDM WITH AND WITHOUT NEUTRINO MASS (768 Mpc^3) – GADGET 2

256 Mpc

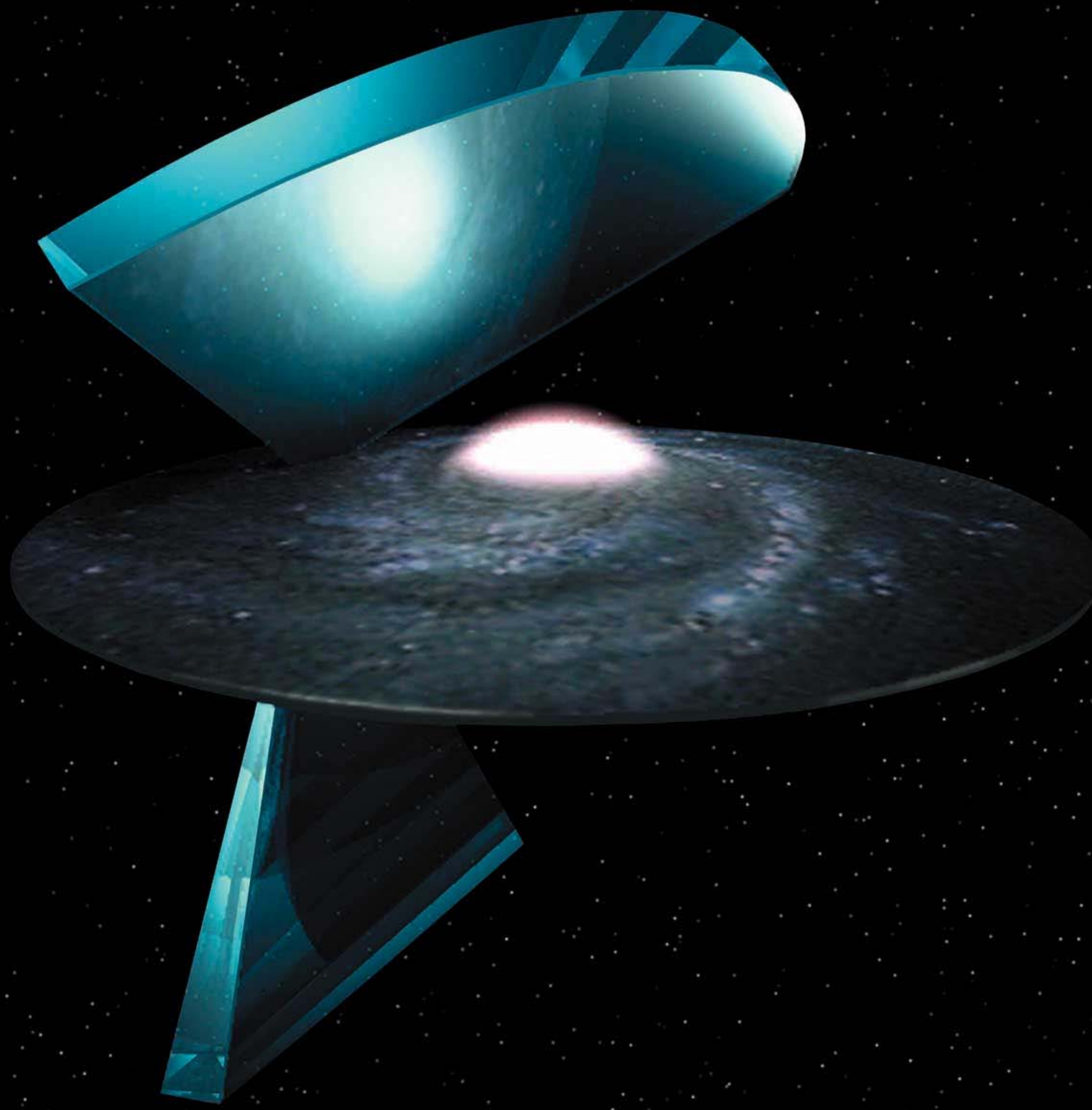


$$\sum m_\nu = 0$$

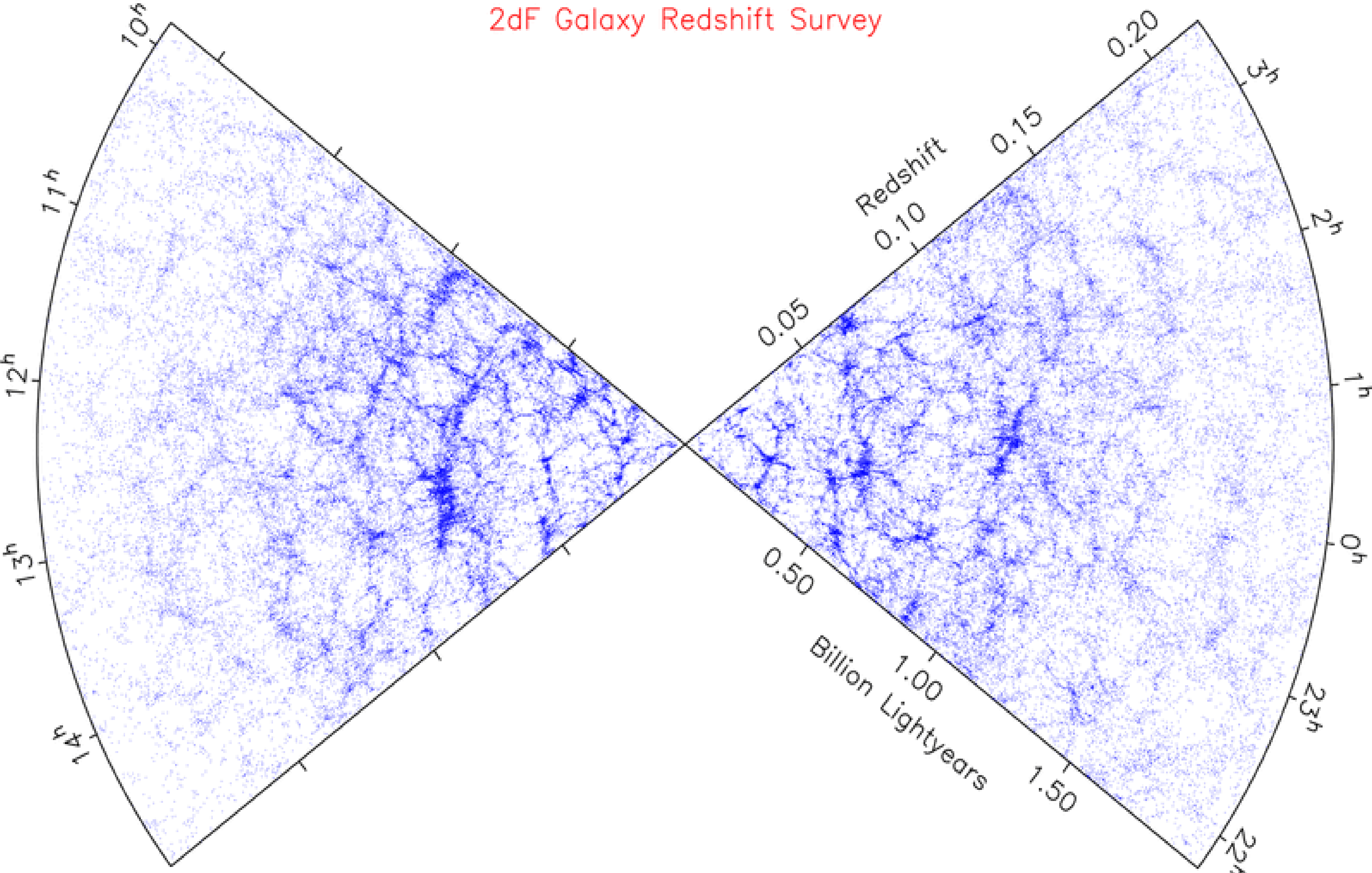
$$\sum m_\nu = 6.9\,\mathrm{eV}$$

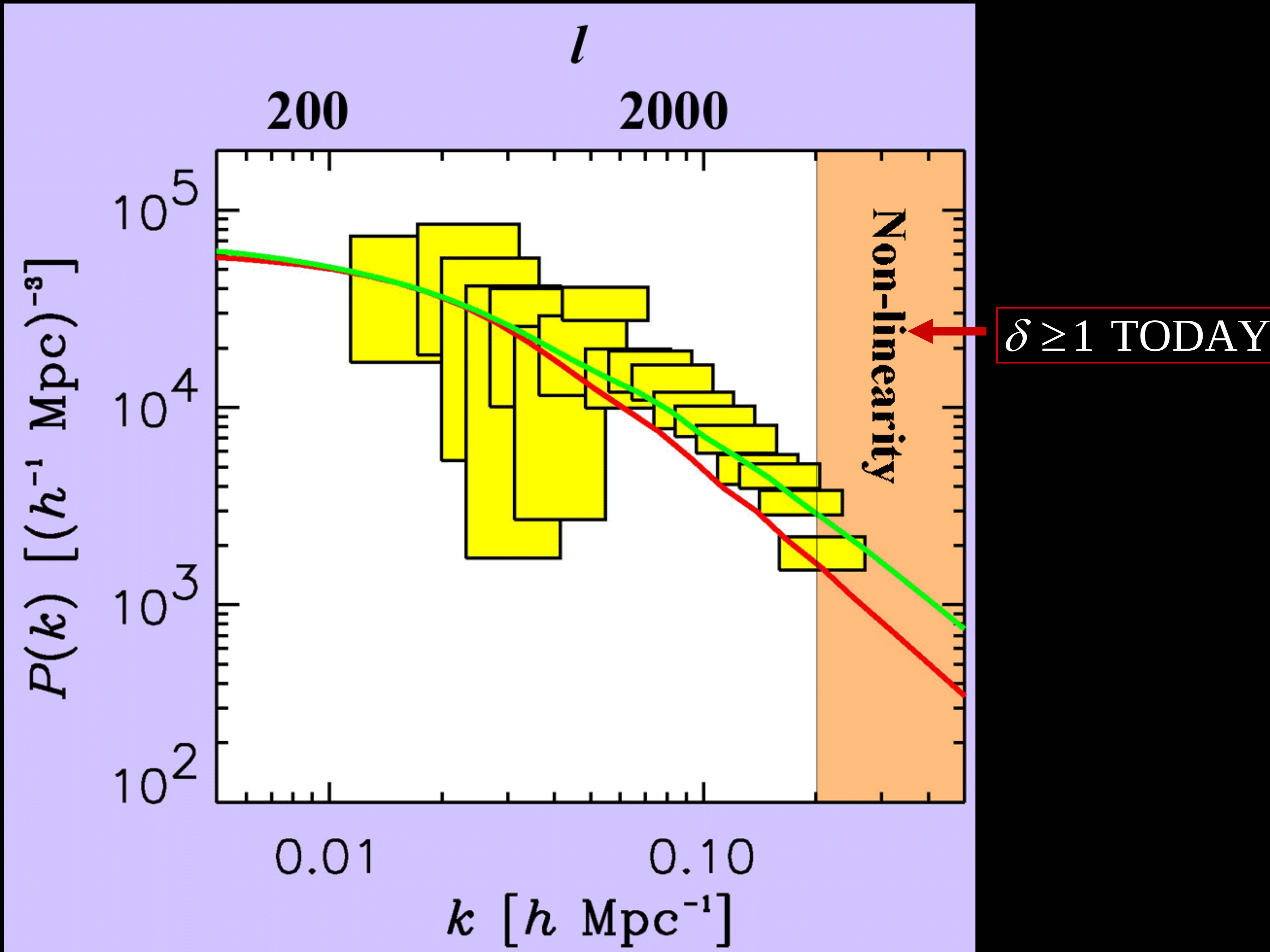
T Haugboelle, University of Aarhus

GALAXY SURVEYS



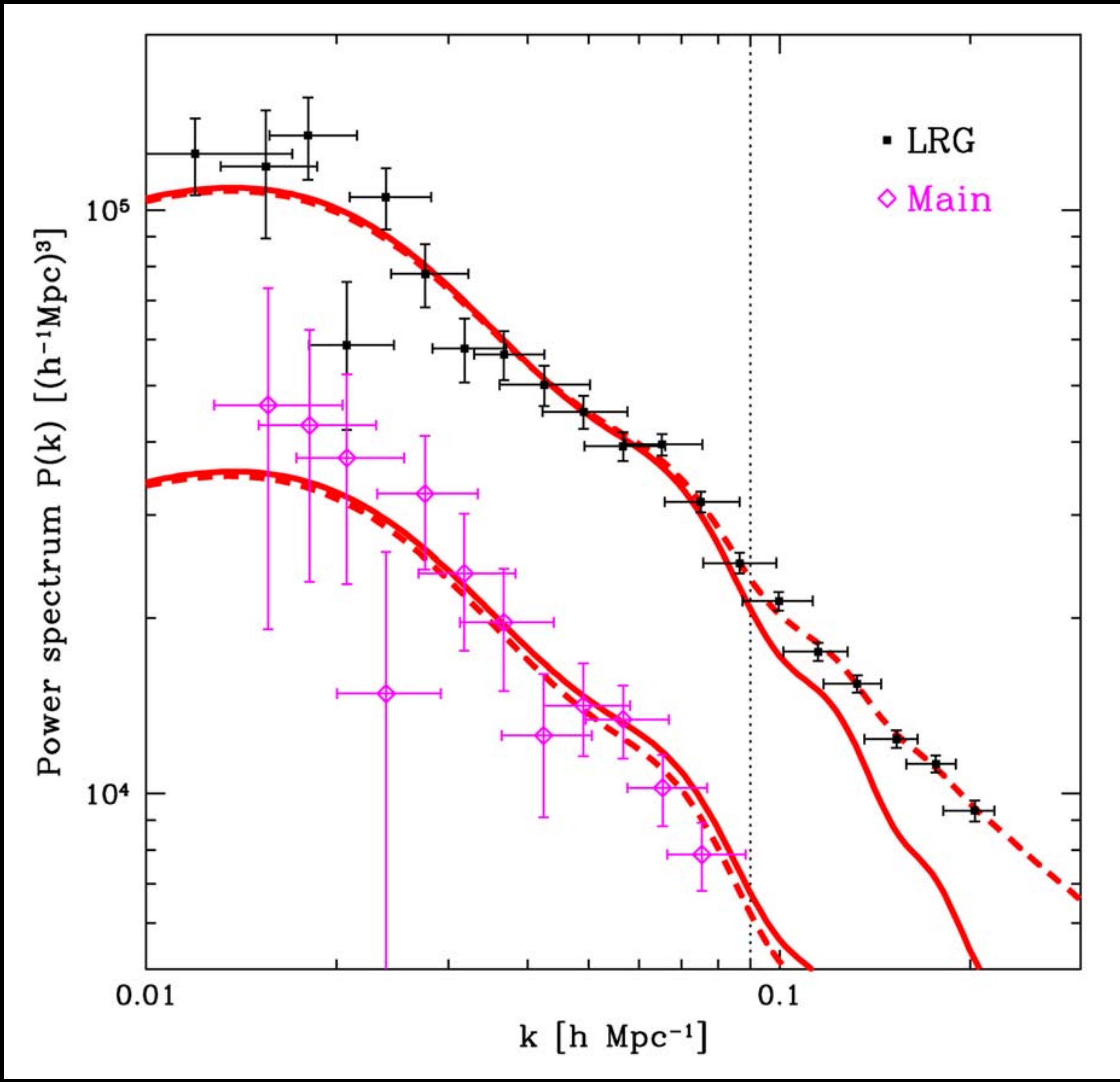
LARGE SCALE STRUCTURE SURVEYS - 2dF AND SDSS





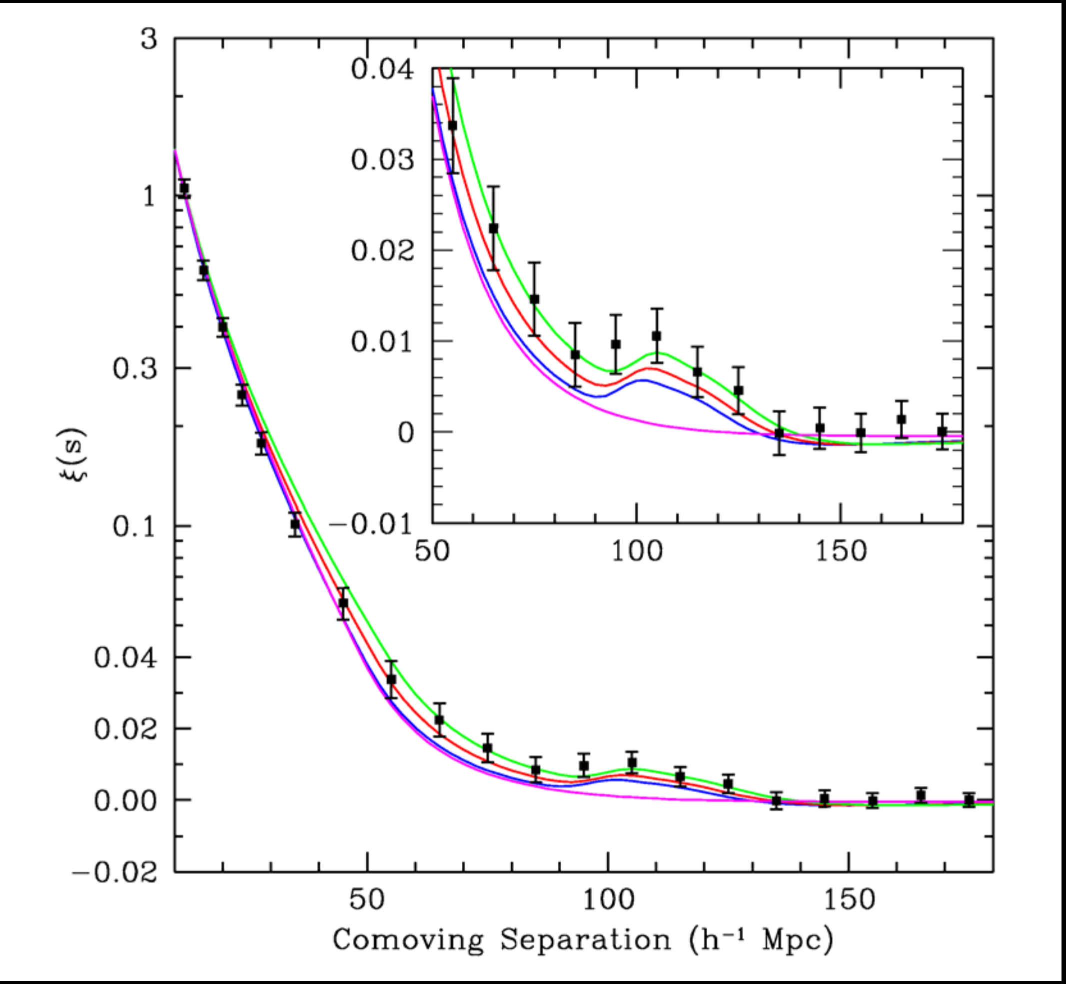
l
200
2000
10^5
10^4
10^3
10^2
$P(k)$ $[(h^{-1}\ \mathrm{Mpc})^{-3}]$
Non-linearity
$\delta \geq 1$ TODAY
0.01
0.10
k $[h\ \mathrm{Mpc}^{-1}]$

SDSS SPECTRUM
TEGMARK ET AL. 2006



astro-ph/0608632

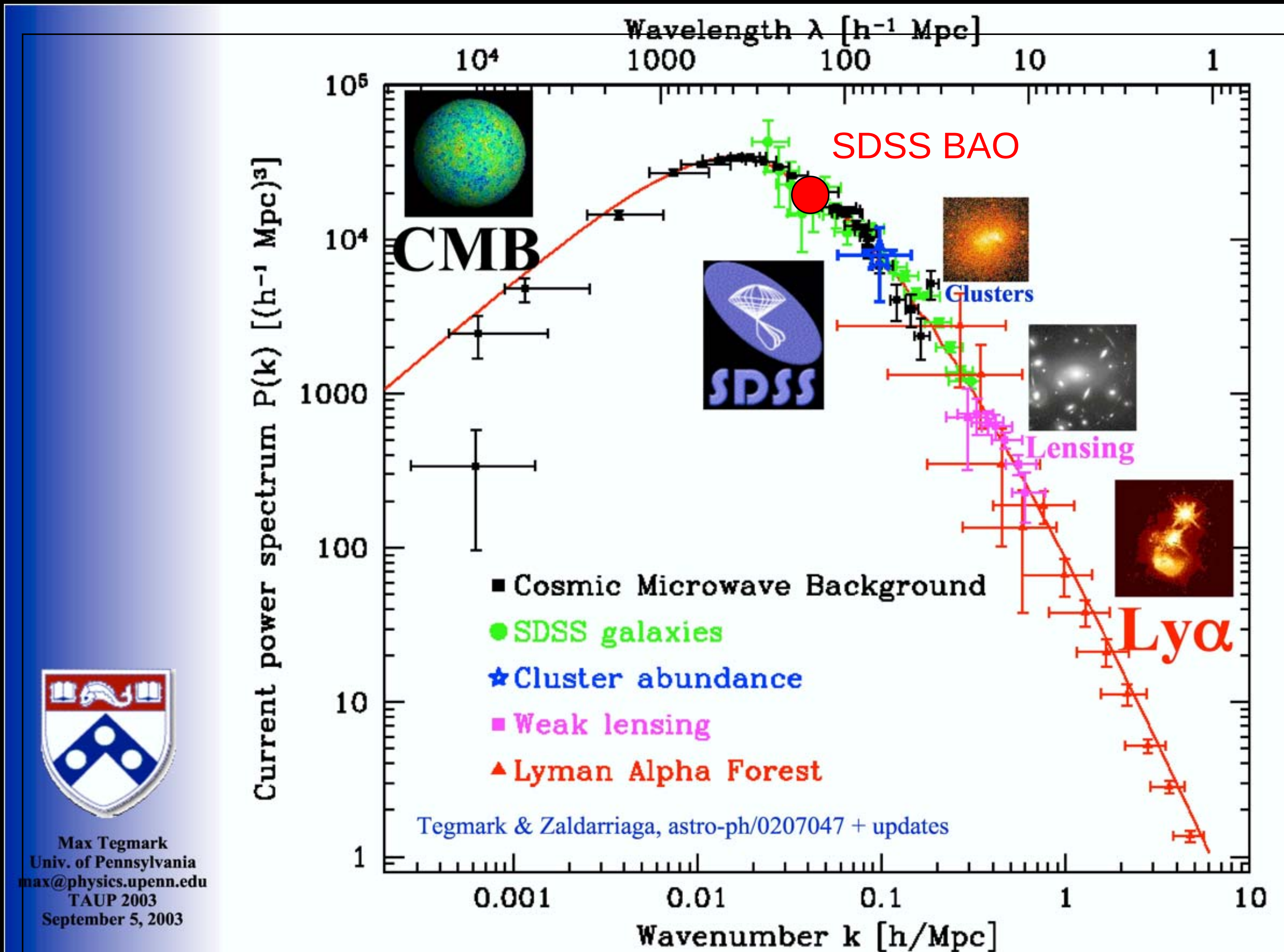
THE SDSS MEASUREMENT OF BARYON OSCILLATIONS IN THE POWER SPECTRUM PROVIDES A FANTASTICALLY PRECISE MEASURE OF THE ANGULAR DISTANCE SCALE AND TURNS OUT TO BE EXTREMELY USEFUL FOR PROBING NEUTRINO PHYSICS



EISENSTEIN ET AL. 2005 (SDSS)

NEUTRINO MASSES ARE THE LARGEST SYSTEMATIC ERROR NOT ACCOUNTED FOR IN THE ANALYSIS

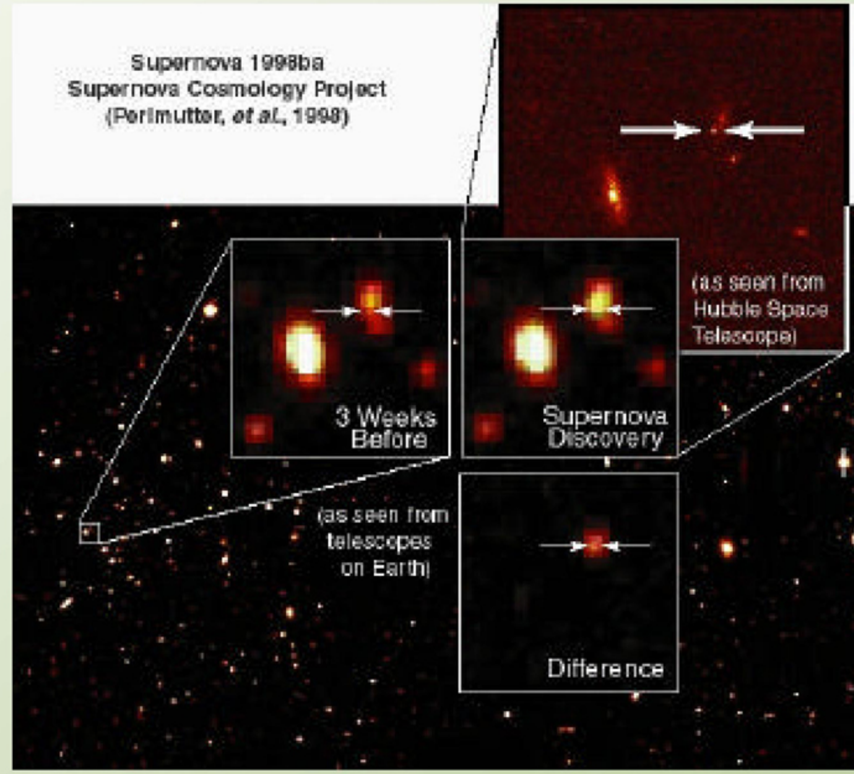
GOOBAR, HANNESTAD, MÖRTSELL, TU 2006

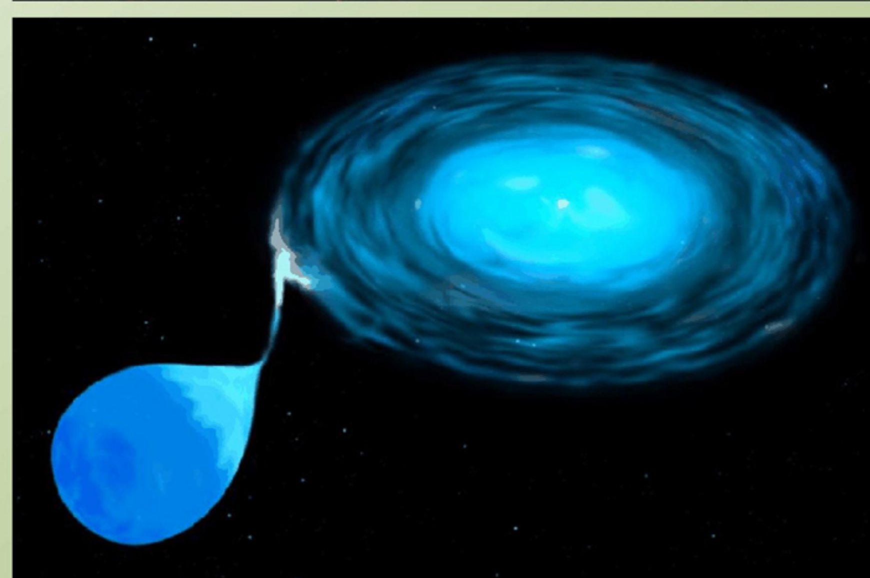


FROM MAX TEGMARK

Supernovae Ia as standard candles

- Supernovae Ia are formed when a white dwarf in a binary system gets heavy enough to burn coal.
- It then explodes, always in the same way and with (roughly) the same luminosity.





Supernovae Ia
How can we learn about the geometry from them?

- Supernovae type Ia are standard candles with an intrinsic luminosity, B. Measure their apparent luminosity, b and determine the luminosity distance, d_L

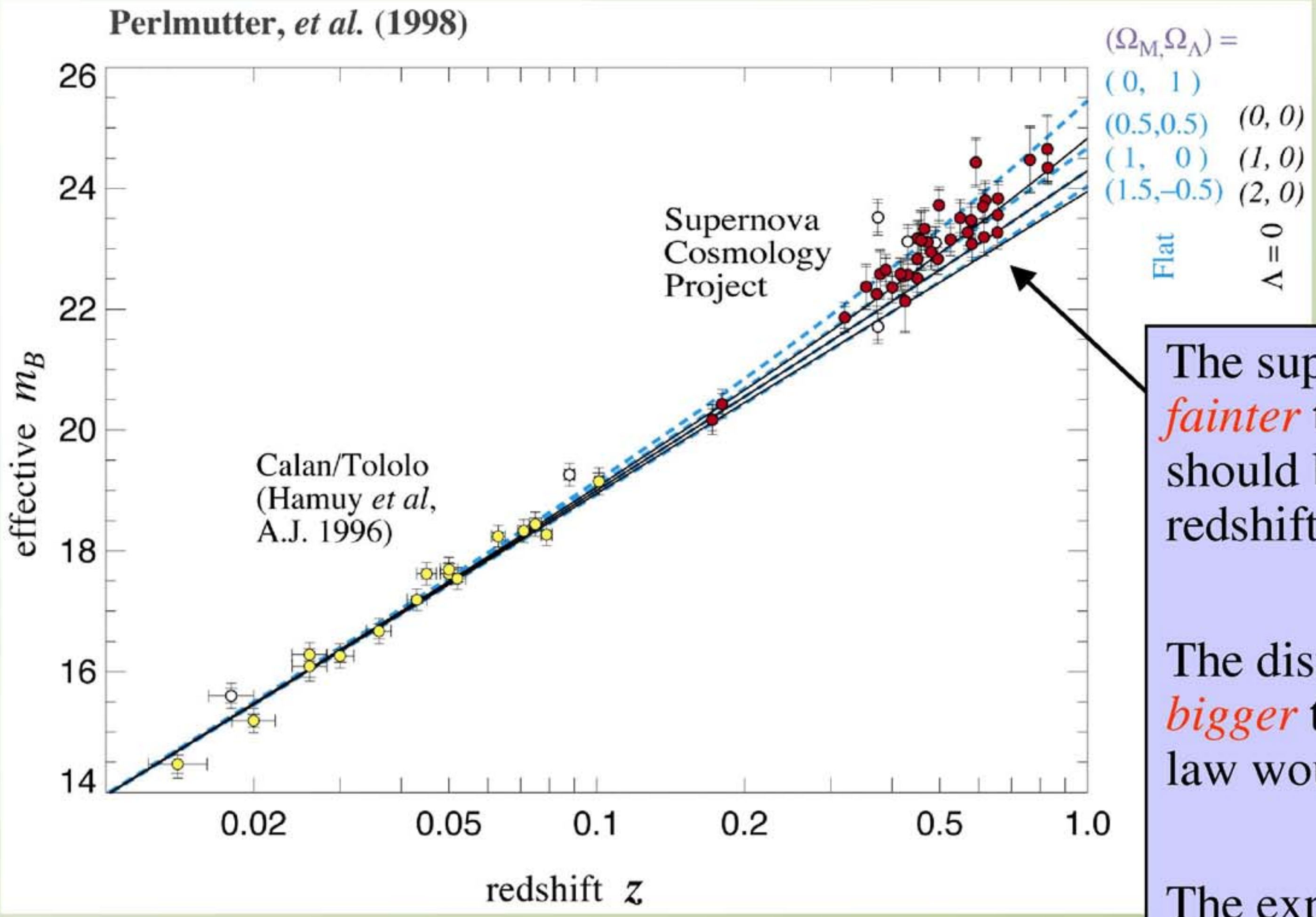
$$b = \frac{B}{4\pi d_L^{\,2}} \qquad \Leftrightarrow \qquad d_L = \sqrt{\frac{B}{4\pi b}}$$

- Or expressed in terms of the redshift, z

$$d_L = \frac{1}{H_0}\left(z + \boxed{f(\Omega_{matter}, \Omega_\Lambda)} z^2 + \ldots\right)$$

Determine the deviation from a linear relationship and we get Ω_{matter} and Ω_Λ.

Supernovae Ia
Hubble diagram



The supernovae are *fainter* than they should be with their redshift.

⇩

The distance is *bigger* than Hubble's law would predict

⇩

The expansion has ***accelerated!***

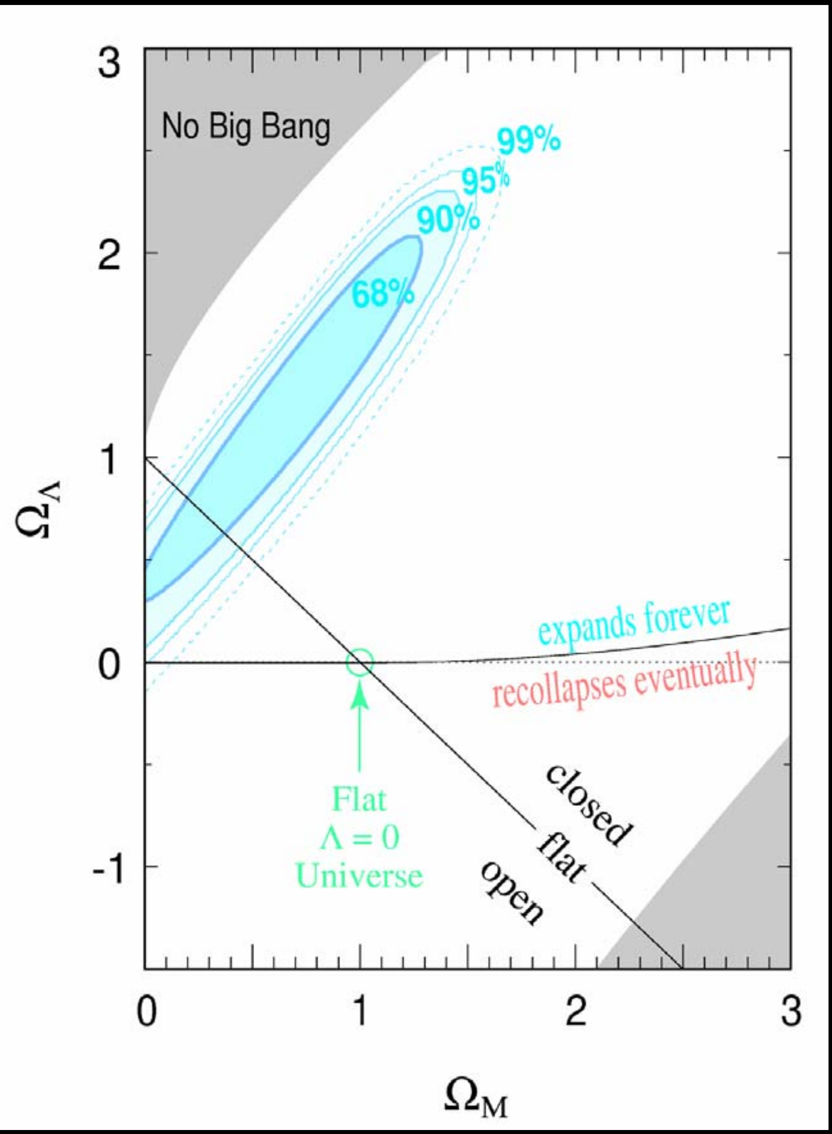
WHAT ARE THE SUPERNOVA OBSERVATIONS ACTUALLY MEASURING?

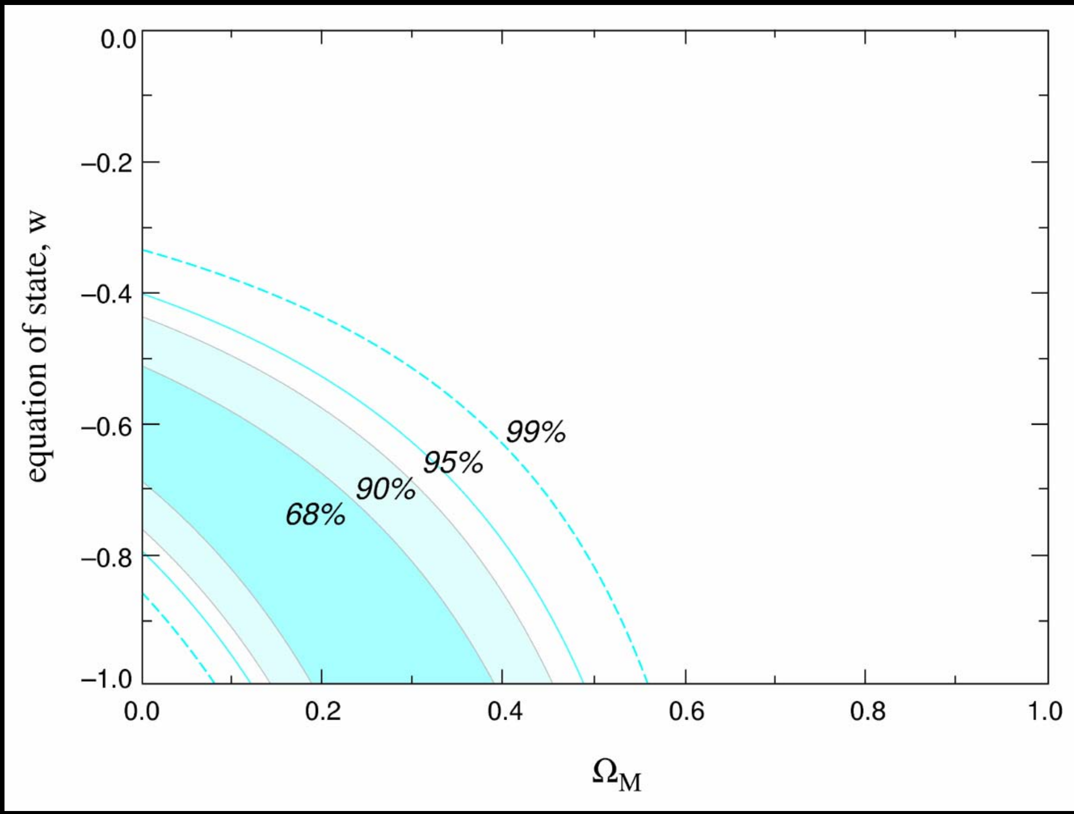
THE DECELERATION PARAMETER

$$q_0 \equiv -\frac{\ddot{a}a}{\dot{a}^2}$$

USING THE FRIEDMANN EQUATION THIS CAN BE CAST AS

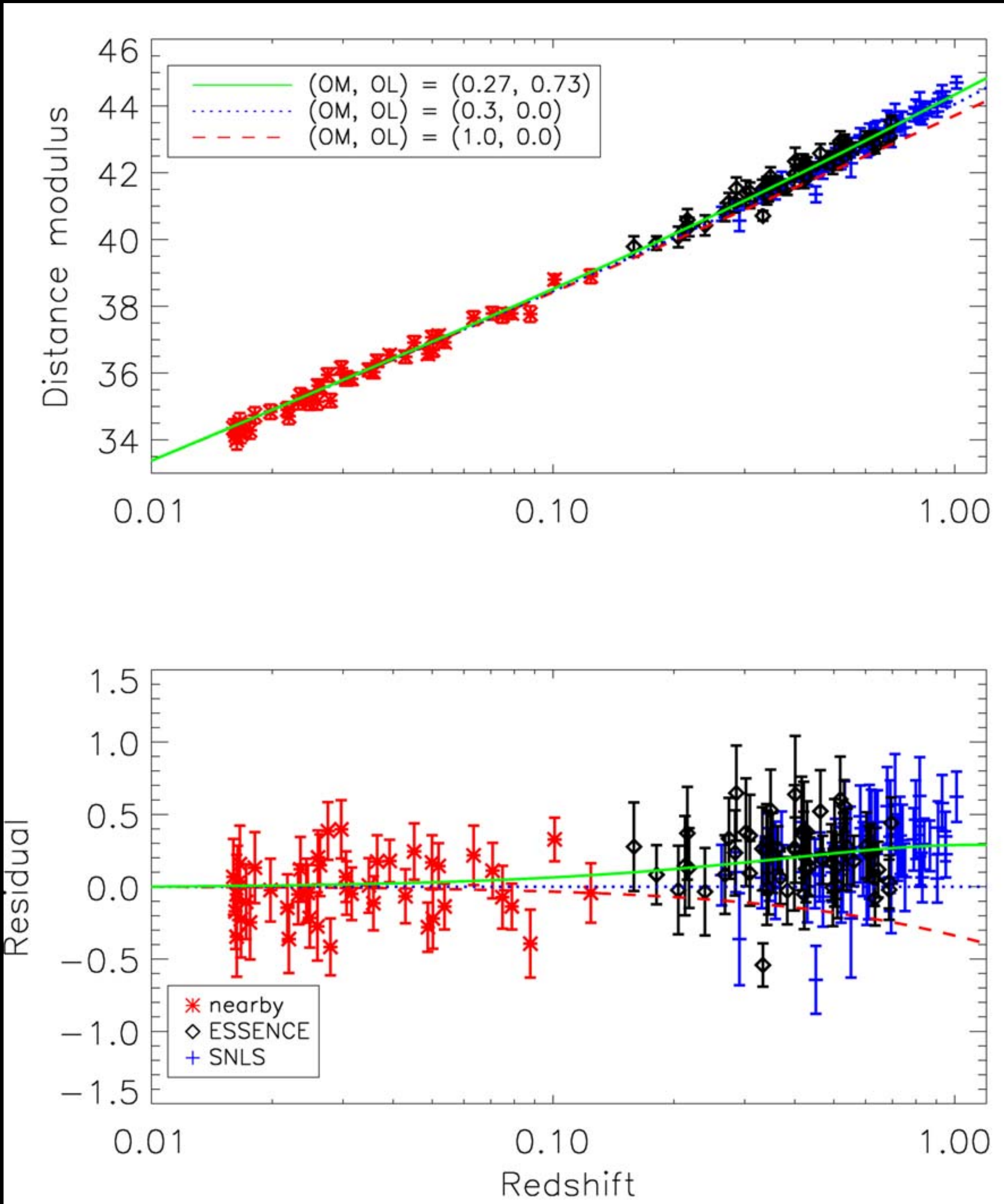
$$q_0 = \frac{1}{2}(\Omega_m - 2\Omega_\Lambda)$$


No Big Bang
99%
95%
90%
68%
expands forever
recollapses eventually
Flat
Λ = 0
Universe
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3


equation of state, w
Ω_M
99%
95%
90%
68%
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.0
0.2
0.4
0.6
0.8
1.0

MOST RECENT DATA FROM "SUPERNOVA LEGACY SURVEY" (SNLS) – OCTOBER 2005

AND THE "ESSENCE" PROJECT – JANUARY 2007



SO, WHAT ARE THE VALUES OF COSMOLOGICAL PARAMETERS GIVEN THE MOST UP-TO-DATE DATA?

Komatsu et al. arXiv:0803.0547 (WMAP-5)