Mössbauer neutrinos

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JHEP **0906** (2009) 049 (arXiv:0904.4346)

MAX-PLANCK-GESELLSCHAFT

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Outline

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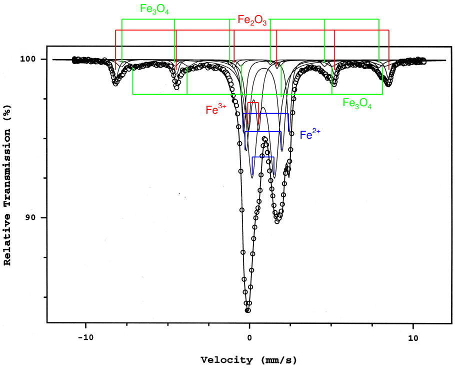

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A similar effect should exist for neutrino emission/absorption in bound state β decay and induced electron capture processes.

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Proposed experiment:

$$\begin{aligned} \text{Production:}\quad & {}^3\mathrm{H} && \to {}^3\mathrm{He}^+ + \bar{\nu}_e + e^-(\text{bound}) \\ \text{Detection:}\quad & {}^3\mathrm{He}^+ + e^-(\text{bound}) + \bar{\nu}_e && \to {}^3\mathrm{H} \end{aligned}$$

^{3}H and ^{3}He are embedded in metal crystals (metal hydrides).

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Physics goals:

- Neutrino oscillations on a laboratory scale: $E = 18.6$ keV, $L^{\text{osc}}_{\text{atm}} \sim 20$ m.
- Gravitational interactions of neutrinos
- Study of solid state effects with unprecedented precision

Mössbauer neutrinos (2)

Mössbauer neutrinos have very special properties:

- Neutrino receives *full* decay energy: $Q = 18.6$ keV
- Natural line width: $\gamma \sim 1.17 \times 10^{-24}$ eV
- Atucal line width: $\gamma \gtrsim 10^{-11}$ eV
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  - Homogeneous broadening (Spin interactions)

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Experimental challenges:

- Is the Lamb-Mössbauer factor (fraction of recoil-free emissions/absorptions) large enough?
- Can a linewidth $\gamma \gtrsim 10^{-11}$ eV be achieved?
- Can the resonance condition be fulfilled?

Mössbauer neutrinos (3)

Recent controversy:

- Does the small energy uncertainty prohibit oscillations of Mössbauer neutrinos?
- Do oscillating neutrinos need to have equal energies resp. equal momenta?

S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G34** (2007) 987, hep-ph/0611285

- Does the time-energy uncertainty relation prevent oscillations?

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⇒ Careful treatment with as few assumptions as possible is needed
⇒ Answer to the above questions will be No.

Outline

Textbook derivation of the oscillation formula

Diagonalization of the mass terms of the charged leptons and neutrinos gives

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} \left(\bar{e}_{\alpha L} \gamma^{\mu} U_{\alpha j} \nu_{jL} \right) W^{-}_{\mu} \; + \; \text{diag. mass terms} \; + \; h.c.$$

(flavour eigenstates: $\alpha = e, \mu, \tau$, mass eigenstates: $j = 1, 2, 3$)

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Assume, at time $t = 0$ and location $\vec{x} = 0$, a flavour eigenstate

$$|\nu(0,0)\rangle = |\nu_\alpha\rangle = \sum_i U^*_{\alpha j} |\nu_j\rangle$$

is produced. At time t and position $\vec{x}$, it has evolved into

$$|\nu(t,\vec{x})\rangle = \sum_i U^*_{\alpha j} e^{-iE_j t + i\vec{p}_j \vec{x}} |\nu_i\rangle$$

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Oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \left| \left\langle \nu_\beta | \nu(t,\vec{x}) \right\rangle \right|^2 = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i(E_j - E_k)t + i(\vec{p}_j - \vec{p}_k)\vec{x}}$$

Equal energies or equal momenta?

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These are *assumptions* or *approximations*, not fundamental principles!

Problems with the textbook derivation

- In general, neither the equal energy assumption nor the equal momentum assumption is physically justified because both violate energy-momentum conservation in the production and detection processes.

R. G. Winter, Lett. Nuovo Cim. **30** (1981) 101
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Energy-momentum conservation for emission of mass eigenstate $|\nu_i\rangle$:

$$E_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\ \mathrm{MeV}$.

To first order in m_i^2:

$$E_i \simeq E + \xi\frac{m_i^2}{2E}\,, \qquad p_i \simeq E - (1-\xi)\frac{m_i^2}{2E}\,, \qquad \xi \approx \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

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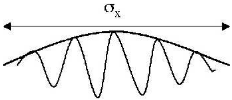
Beuthe, Giunti, Grimus, Kiers, Kim, Lee, Mohanty, Nussinov, Stockinger, Weiss, ...

Conditions for oscillations in a wave packet approach



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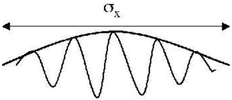
- Coherence in production and detection processes



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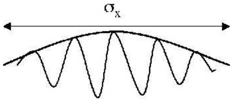
  Neutrino oscillations are caused by the superposition of different mass eigenstates.

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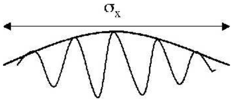

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$$\sigma_m^2 = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2} > \Delta m^2$$

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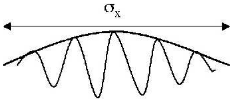

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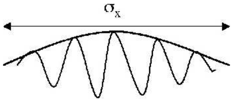
This is easily fulfilled for Mössbauer neutrinos, since

$$\sigma_E \sim 10^{-11}\ \text{eV}$$
$$\sigma_p = 1/2\sigma_x \sim 1/\text{interatomic distance} \sim 10\ \text{keV}$$
$$E = p = 18.6\ \text{keV}$$

Conditions for oscillations in a wave packet approach

- Coherence in production and detection processes
- Coherence maintained during propagation



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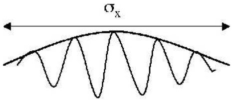

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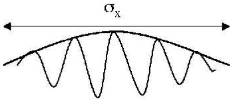

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It can be shown that, for Mössbauer neutrinos, σ_p is small enough, so that

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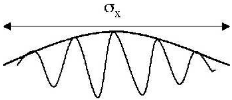

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$$L^{\rm osc} \ll L^{\rm coh}.$$

$\Rightarrow$ Stanard oscillation formula is approximately recovered:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\Big[-2\pi i \frac{L}{L^{\rm osc}_{jk}}\Big]$$

$$L^{\rm osc}_{jk} = \frac{4\pi E}{\Delta m^2_{jk}}$$

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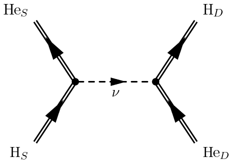

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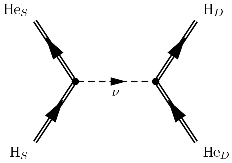

External particles reside in harmonic oscillator potentials.
E.g. for ^{3}H atoms in the source:

$$\psi_{\mathrm{H},S}(\vec{x},t) = \left[\frac{m_\mathrm{H}\omega_{\mathrm{H},S}}{\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_\mathrm{H}\omega_{\mathrm{H},S}|\vec{x}-\vec{x}_S|^2\right]\cdot e^{-iE_{\mathrm{H},S}t}$$

Oscillation amplitude

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2 \left(\frac{m_{\rm H}\omega_{{\rm H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\rm H}\omega_{{\rm H},S}|\vec{x}_1 - \vec{x}_S|^2\right] e^{-iE_{{\rm H},S}t_1} \\
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\cdot \left(\frac{m_{\rm H}\omega_{{\rm H},D}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\rm H}\omega_{{\rm H},D}|\vec{x}_2 - \vec{x}_D|^2\right] e^{+iE_{{\rm H},D}t_2} \\
\cdot \sum_j \mathcal{M}^{\mu}\mathcal{M}^{\nu *}|U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)} \\
\cdot \bar{u}_{e,S}\gamma_\mu(1-\gamma^5)\, \frac{i(\not{p}+m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon}\, (1+\gamma^5)\gamma_\nu u_{e,D}.
\end{aligned}
$$

Oscillation amplitude

$$
\begin{aligned}
i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2 \left(\frac{m_{\mathrm{H}}\omega_{\mathrm{H},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\mathrm{H}}\omega_{\mathrm{H},S}|\vec{x}_1-\vec{x}_S|^2\right] e^{-iE_{\mathrm{H},S}t_1} \\
\cdot \left(\frac{m_{\mathrm{He}}\omega_{\mathrm{He},S}}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_{\mathrm{He}}\omega_{\mathrm{He},S}|\vec{x}_1-\vec{x}_S|^2\right] e^{+iE_{\mathrm{He},S}t_1} \\
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Evaluation:

- $dt_1\, dt_2$-integrals $\rightarrow$ energy-conserving δ functions $\rightarrow$ p_0-integral trivial
- $d^3x_1\, d^3x_2$-integrals are Gaussian
- d^3p-integral: Use Grimus-Stockinger theorem

The Grimus-Stockinger theorem

Let $\psi(\vec{p})$ be a three times continuously differentiable function on $\mathbb{R}^3$, such that ψ itself and all its first and second derivatives decrease at least like $1/|\vec{p}|^2$ for $|\vec{p}| \to \infty$. Then, for any real number $A > 0$,

$$\int d^3p \, \frac{\psi(\vec{p})\, e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}|\to\infty} -\frac{2\pi^2}{L}\psi(\sqrt{A}\tfrac{\vec{L}}{L})e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}}).$$

$\Rightarrow$ Quantification of requirement of on-shellness for large $L = |\vec{L}|$.

W. Grimus, P. Stockinger, Phys. Rev. **D54** (1996) 3414, hep-ph/9603430

From the amplitude to the transition rate

Amplitude:

$$i\mathcal{A} = \frac{-i}{2L}\mathcal{N}\,\delta(E_S - E_D)\,\exp\left[-\frac{E_S^2 - m_j^2}{2\sigma_p^2}\right]\sum_j \mathcal{M}^\mu \mathcal{M}^{\nu *}|U_{ej}|^2\, e^{i\sqrt{E_S^2 - m_j^2}L}$$

$$\cdot\, \bar{u}_{e,S}\gamma_\mu \tfrac{1-\gamma^5}{2}(\not{p}_j + m_j)\tfrac{1+\gamma^5}{2}\gamma_\nu u_{e,D},$$

$$\sigma_p^{-2} = (m_{\mathrm{H}}\omega_{\mathrm{H},S} + m_{\mathrm{He}}\omega_{\mathrm{He},S})^{-1} + (m_{\mathrm{H}}\omega_{\mathrm{H},D} + m_{\mathrm{He}}\omega_{\mathrm{He},D})^{-1}$$

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Transition rate: Integrate $|\mathcal{A}|^2$ over densities of initial and final states

$$\Gamma \propto \int_0^\infty dE_{\mathrm{H},S}\, dE_{\mathrm{He},S}\, dE_{\mathrm{He},D}\, dE_{\mathrm{H},D}$$

$$\cdot\,\delta(E_S - E_D)\rho_{\mathrm{H},S}(E_{\mathrm{H},S})\,\rho_{\mathrm{He},D}(E_{\mathrm{He},D})\,\rho_{\mathrm{He},S}(E_{\mathrm{He},S})\,\rho_{\mathrm{H},D}(E_{\mathrm{H},D})$$

$$\cdot \sum_{j,k}|U_{ej}|^2|U_{ek}|^2\, \underbrace{\exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right]}_{\text{Analogue of Lamb-Mössbauer factor (Recoil-free fraction)}}\, \underbrace{e^{i\left(\sqrt{E_S^2-m_j^2}-\sqrt{E_S^2-m_k^2}\right)L}}_{\text{Oscillation phase}}$$

The Lamb-Mössbauer factor

The Lamb-Mössbauer factor is the relative probability of recoil-free emission and absorption, compared to the total emission and absorption probability.

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Convenient reformulation:

$$\exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right] = \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

where $(p_{jk}^{\min})^2 = E_S^2 - \max(m_j^2, m_k^2)$.

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⇒ Localization condition

$$4\pi\sigma_x E/\sigma_p \lesssim L_{jk}^{\rm osc},$$

(with $\sigma_x = 1/2\sigma_p$) is satisfied if $L_{jk}^{\rm osc} \gtrsim 2\pi\sigma_x$, which is easily fulfilled in realistics situations.

Line broadening

Energy levels of ^3H and ^3He in the source and detector are smeared e.g. due to spin-spin interactions, crystal impurities, lattice defects, etc.

R. S. Raghavan, hep-ph/0601079

W. Potzel, Phys. Scripta **T127** (2006) 85

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$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$

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Result for two neutrino flavours:

$$\Gamma \propto \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S+\gamma_D)^2}{4}} \cdot \left\{ 1 - 2s^2c^2 \left[1 - \frac{1}{2}(e^{-L/L_S^{\mathrm{coh}}} + e^{-L/L_D^{\mathrm{coh}}}) \cos\left(\pi \frac{L}{L^{\mathrm{osc}}}\right) \right] \right\}$$

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In realistic cases: $L_{S,D}^{\mathrm{coh}} \gg L^{\mathrm{osc}} \Rightarrow$ Decoherence is not an issue.

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  - Nonzero line width leads to coherence condition.
  - Both conditions are easily fulfilled in realistic experiments.

Thank you!